Starred questions are suitable for students aiming for a credit or higher.

1. Translate the following into symbolic statements using quantifiers:

   (a) Australians are from the southern hemisphere.

   (b) No-one living in Europe lives in the southern hemisphere.

   (c) No Australian lives in Europe.

   Decide whether (c) follows logically from (a) and (b). Clearly (c) is false, so where might be the source of confusion?

2. Translate the following (attributed to Abraham Lincoln) into symbolic statements using quantifiers:

   (a) You can fool some of the people all of the time.

   (b) You can fool all of the people some of the time.

   (c) You can’t fool all of the people all of the time.

   Decide whether (a), (b) and (c) taken together are logically consistent.

3. Use the rules of deduction of the Predicate Calculus to prove the following sequents:

   (a) $(\forall x)(F(x) \Rightarrow G(x)) \vdash (\forall x)F(x) \Rightarrow (\forall x)G(x)$

   (b) $(\forall x)(F(x) \Rightarrow \sim G(x)), (\forall x)(H(x) \Rightarrow G(x)) \vdash (\forall x)(F(x) \Rightarrow \sim H(x))$

   *(c) $(\forall x)((F(x) \lor G(x)) \Rightarrow H(x)), (\forall x)\sim H(x) \vdash (\forall x)\sim F(x)$

   *(d) $(\forall x)(G(x) \Rightarrow H(x)), (\exists x)(G(x) \land F(x)) \vdash (\exists x)(F(x) \land H(x))$

   *(e) $(\exists x)(G(x) \land \sim H(x)), (\forall x)(G(x) \Rightarrow F(x)) \vdash (\exists x)(F(x) \land \sim H(x))$

   (f) $(\forall x)(F(x) \land G(x)) \vdash (\forall x)F(x) \land (\forall x)G(x)$

   (g) $(\forall x)F(x) \land (\forall x)G(x) \vdash (\forall x)(F(x) \land G(x))$

   *(h) $(\exists x)(F(x) \lor G(x)) \vdash (\exists x)F(x) \lor (\exists x)G(x)$

   *(i) $(\exists x)F(x) \lor (\exists x)G(x) \vdash (\exists x)(F(x) \lor G(x))$

   *(j) $(\forall x)F(x) \lor (\forall x)G(x) \vdash (\forall x)(F(x) \lor G(x))$

4. Explain why proofs for the converses to parts (a) and (j) of the previous exercise cannot exist, assuming a Soundness Metatheorem for the Predicate Calculus.