Assignment 1

Due: Friday September 17, 4pm

Formulate Questions 1 and 2 as linear programming problems, giving full details. Solve Question 1 graphically. Solve Questions 2 and 3 using the simplex algorithm. Do Question 2 by hand. You must use Matlab (but not the lp command) to do Question 3. With Question 3, do not write out what you have done on the computer, just hand in printout(s) of it, minus any errors and blank lines which should be edited out. Each of your computer printouts must contain the banner indicating your username. Do Question 4 by hand. You must staple your assignment in a manila folder, clearly labelled with your name and SID number on the front. Place it in the bottom hand-in boxes outside Room 623 on Level 6 of Carslaw, by 4pm on the due date. This assignment is worth a total of 10% toward your final mark.

1. A tea company packages and sells tea under a “premium label” and a “regular label”. Both are blended from three types of tea leaves $T_1$, $T_2$ and $T_3$, purchased in bulk. The regular label is blended from $T_1$, $T_2$ and $T_3$ in the proportions 50%, 30% and 20% by weight. The premium label is blended from $T_1$, $T_2$ and $T_3$ in the proportions 15%, 25% and 60%. The company can sell its product for $13.00 per kg for regular label and $19.00 per kg for premium label. One week the company is given the option of buying up to 50 kg of $T_1$ at $8 per kg, up to 30 kg of $T_2$ at $12 per kg and up to 25 kg of $T_3$ at $16 per kg. Calculate the profit per kg of each blend of tea. How much of each tea blend should the distributor produce to maximize its profit? Interpret your answers fully.

2. A company manufactures three products, A, B and C at its plant. The parts of each product must be manufactured, the products are then assembled and finally the finished products packaged for distribution to wholesalers. Each unit of product A takes 8 hours to manufacture, 3 hours to assemble and 1 hour 30 minutes to package. The corresponding times for product B are 5 hours, 2 hours and 1 hour 30 minutes, and for product C, 10 hours, 4 hours and 2 hours. The manufacturer has up to 600 hours of manufacturing time per week using skilled labour, 220 hours of assembling time, which uses semi-skilled labour, and 120 hours of packaging time which uses unskilled labour. If the profits per unit A, B and C are $230, $120 and $260, respectively, how many units of each product should the manufacturer produce, assuming all product can be sold?
3. By introducing appropriate variables put the following problem into a form suitable for the two-phase simplex algorithm. Hence write down a starting solution for phase one, and solve the problem using the MATLAB routines introduced in Tutorial 4.

Max \( Z = 3x_1 + 4x_2 \)

subject to

\[
\begin{align*}
3x_1 + x_2 & \leq 4 \\
x_1 + 3x_2 & = 7 \\
x_1 + 2x_2 & \geq 3 \\
\end{align*}
\]

with \( x_1 \geq 0, \ x_2 \geq 0 \).

4. The Daily Planet prints its daily paper at three plants and distributes them to four distribution centres. The three plants print 40,000, 50,000 and 30,000 papers, respectively, daily. The demands for newspapers at the distribution centres 1, 2, 3, 4 are 30,000, 20,000, 30,000 and 40,000, respectively. The net cost of transporting papers from plant \( i \) to distribution centre \( j \) is given by the following table in units of cents per 1000 papers.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Distribution Centre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>53</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
</tr>
</tbody>
</table>

(a) How many papers should be trucked from each plant to each distribution centre to minimize the total transport cost? What is the minimum cost? Formulate the problem as a transportation problem and then use the transportation algorithm to find the optimal solution.

(b) Formulate the problem as a linear programming problem. (Do not solve it.)