Google: MATH1111

- Click on "Worksheet for Monday and Tuesday"
- Then "Solutions"

Sometimes number of significant figures can be ambiguous,

e.g. 76,420 is unambiguous (5 s.f.)

but 76,420 = \[ 7.642 \times 10^4 \] if 4 significant
\[ 7.642 \times 10^4 \] if 0 not significant

Rule with multiplication: the least number of significant figures used in the data should be quoted for final answers.

Operations on sets (useful for expressing solutions)
- union \( A \cup B \)
- intersection \( A \cap B \)
- slash \( A \setminus B \)
Venn diagram method:

\[(A \cup B) \cap C = \{2, 4, 6\}\]

\[A \cap C = \]

\[B \cap C = \]

\[(A \cap C) \cup (B \cap C) = \]

\[= (A \cup B) \cap C.\]
**General fact:** \((A \cup B) \cap C = (A \cap C) \cup (B \cap C)\)

Like distributivity in arithmetic.

**Example:** \((2 + 3) \times 4 = (2 \times 4) + (3 \times 4)\) \(= 8 + 12 = 20\)

\(20 = \frac{5 \times 4}{1}\)

**Exercise:** Use Venn diagrams to check also

\((A \cap B) \cup C = (A \cup C) \cap (B \cup C)\)

But in ordinary arithmetic

\((2 \times 3) + 4 \neq (2 + 4) \times (3 + 4)\) \(= 6 \times 7 = 42\)

(set theory arithmetic "mines" than real number arithmetic)

**Solution sets of equations & inequalities:**

**Example** (Q3(i) or Thru Fi) Solve \((x-1)(x-2) = 0\)

**Solution:** \((x-1)(x-2) = 0 \Rightarrow x-1 = 0 \text{ or } x-2 = 0\)

\(\Rightarrow x = 1 \text{ or } x = 2\)

Solution set = \(\{1, 2\}\)
Useful fact (reason why we obsess about factorising expressions):

\[ ab = 0 \implies a = 0 \text{ or } b = 0 \]

Equivalent to

\[ a \neq 0 \text{ and } b \neq 0 \implies ab \neq 0 \]

Reason: \( ab \) represents the signed area of the rectangle.

Thought of as positive if \( a \cdot b > 0 \) \( \text{or } a \cdot b < 0 \) both positive \( \text{or } \) both negative.

Or negative if \( a \cdot b < 0 \) \( \text{or } a \cdot b > 0 \) one positive \( \text{and } \) other negative.

Positive real numbers:

\[ -1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow \]
\[ 2 \times 2 = 4, \quad (-2) \times 2 = 2 \times (-2) = -4 \]

but
\[ (-2) \times (-2) = -(2 \times (-2)) = -(-2) = 4 \]

\[ \text{Example (inequality): solve} \]
\[ (x-1)(x-2) > 0 \]

[Some exploration: put]
\[ f(x) = (x-1)(x-2) \]

Find
\[ f(0) = (0-1)(0-2) = (-1)(-2) = 2 \]
\[ f(1) = (1-1)(1-2) = 0 \times (-1) = 0 \]
\[ f(-1) = (-1-1)(-1-2) = (-2)(-3) = 6 \]
\[ f(2) = (2-1)(2-2) = 1(0) = 0 \]
\[ f(-2) = (-2-1)(-2-2) = (-3)(-4) = 12 \]
\[ f\left(\frac{3}{2}\right) = \left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right) = \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4} \]
When do we have \((x-1)(x-2) > 0\)?

Solution using a sign diagram:

\[
\begin{array}{c|c|c|c|c|c}
& 1 & 2 & \\
(x-1)(x-2) & - & + & 0 & + & \\
\end{array}
\]

Interval:

\((-\infty, 1)\) \quad \text{and} \quad (2, \infty)

Answer: \((x-1)(x-2) > 0\) when \(x < 1\) or \(x > 2\)

Solution set: \(\{x \in \mathbb{R} \mid x < 1 \text{ or } x > 2\}\)

Interval notation:

\((-\infty, 1) \cup (2, \infty)\)
\[ a \quad b \]
\[ [a, b] = \{ x \in \mathbb{R} \mid a \leq x \leq b \} \]
\[ (a, b) = \{ x \in \mathbb{R} \mid a < x < b \} \]
\[ (a, b] = \{ x \in \mathbb{R} \mid a < x \leq b \} \]
\[ (-\infty, a] = \{ x \in \mathbb{R} \mid x \leq a \} \]
\[ (-\infty, a) = \{ x \in \mathbb{R} \mid x < a \} \]
\[ [b, \infty) = \{ x \in \mathbb{R} \mid b \leq x \} \]
\[ (b, \infty) = \{ x \in \mathbb{R} \mid b < x \} \]
\[ \mathbb{R} = (-\infty, \infty) \]