A1. Complete the following table giving the number of selections of \( m \) objects from a set of size \( n \) in the various cases:

<table>
<thead>
<tr>
<th>selection</th>
<th>ordered</th>
<th>unordered</th>
</tr>
</thead>
<tbody>
<tr>
<td>with repetition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>without repetition</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find the following numbers, expressing each final answer as a single integer. (Include brief working if you wish.)

(i) The number of ordered sequences of 3 digits where the first digit is not 0 but repetitions are allowed:

**Answer:**

(ii) The number of ordered sequences of 3 digits without repetition where the first digit is not 0:

**Answer:**

(iii) The number of ways, unordered without repetition, of choosing 3 balls from 10 balls:

**Answer:**

(iv) The number of ways, unordered with repetition, of choosing 3 balls from an unlimited supply of balls in 10 different colours:

**Answer:**

[10 marks]
A2. Consider each of the following statements. Circle T if you believe the statement is true. Circle F if you believe the statement is false. (Simple guessing is inadvisable. Marks may be deducted for more than three incorrect answers.)

(i) If $A$ and $B$ are finite sets then $|A \cap B| = |A| + |B| - |A \cup B|$. T F

(ii) If there are 80 students in the class then at least 4 have surnames beginning with the same letter. T F

(iii) If $A$, $B$, $C$ are any sets then $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$. T F

(iv) There exist sets $A$, $B$ and $C$ such that $|A| = 5$, $|B| = 4$, $|C| = 3$, $|A \cup B \cup C| = 10$, $|A \cap B| = 2$ and $|A \cap B \cap C| = 1$. T F

(v) To say that a set $X$ of integers has a largest element is to say $(\forall x \in X)(\exists m \in X) x \leq m$. T F

(vi) The negation of $(\forall x)(\exists y) P(x, y) \Rightarrow Q(x, y)$ is $(\exists x)(\forall y) P(x, y) \land \sim Q(x, y)$. T F

(vii) MERGESORT is a divide and conquer algorithm. T F

(viii) BUBBLESORT has running time $O(N \log N)$. T F

(ix) If $N$ is any integer then $N = \lfloor N/2 \rfloor + \lceil N/2 \rceil$. T F

(x) The Euclidean Algorithm is a divide and conquer algorithm. T F

(xi) The g.c.d. of two consecutive Fibonacci numbers is 1. T F

(xii) The equation $4x = 3 (\mod 9)$ has no integer solution for $x$. T F

(xiii) The system of equations $\begin{cases} x = 0 \pmod{8} \\ x = 10 \pmod{7} \end{cases}$ has a unique solution for $x \in \mathbb{Z}_{56}$. T F

(xiv) Fermat’s Little Theorem implies that if $x$ is an integer not divisible by a prime $p$ then $x^p = x \pmod{p}$. T F

(xv) A recurrence relation $a_n = ra_{n-1} + sa_{n-2} + f(n)$ is homogeneous if $f(n) \neq 0$. T F

(xvi) If $c_n$ is the complementary function and $p_n$ is a particular solution of a recurrence $a_n = ra_{n-1} + sa_{n-2} + f(n)$, then the general solution is $a_n = c_n + p_n$. T F

(xvii) The sequence 0, 1, 2, 3, . . . has generating function $\sum_{n=0}^{\infty} nz^{n+1}$. T F

(xviii) If $\sum_{n=0}^{\infty} a_n z^n = \frac{1}{1 - 3z + 2z^2}$ then $a_2 = 9$. T F

(xix) If $\begin{vmatrix} 1 & 1 & 1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} > 0$ and $P = (x_1, x_2)$, $Q = (y_1, y_2)$, $R = (z_1, z_2)$, then the triangle $\triangle PQR$ is oriented clockwise. T F

(xx) $(3, 5)$ lies outside the triangle with vertices $(0, 0)$, $(4, 0)$, $(2, 7)$. T F

[10 marks]

THIS IS THE END OF SECTION A