Welcome to MATH 3005 (LOGIC), Semester 1, 2005

Course Outline: We present an introduction to modern Mathematical Logic.

After discussing what might be meant by a routine computational procedure, we introduce the concept of a Turing Machine, and discover that, though very simple, Turing machines are capable in theory of simulating much of the routine computational work which seems to play a large part in mathematics. However, we find that there are certain mathematical tasks which cannot be performed by a Turing machine, nor by any finite state machine we seem to be able to imagine. We show that the famous “Blank Tape Halting Problem” (BTHP) is one of these “Turing impossible” tasks.

We then study Logical Systems and discuss whether it is possible to set up such a system in such a way that it becomes routine to discover and prove mathematical truths, and to disprove mathematical falsehoods. We find that, because of the BTHP, there is no way such a system can be set up, if it is to include any substantial mathematics (such as elementary number theory). We conclude with a discussion of one of the most dramatic discoveries of 20th century mathematics, Gödel’s Incompleteness Theorem, and its relationship with the BTHP.

Text: MATH 3005 (LOGIC) COURSE NOTES 2005, by H.M.Gastineau-Hills. Available at KOPY STOP, 55 Mountain Street, Broadway.

Lectures: Two per week.

Tutorials: Twelve, one per week, starting week 2. Participation marks may be earned, one per week, to a maximum of 10. Each tutorial sheet will be handed out in lectures the week before (also obtainable from Carslaw 527 and from the Web — see below). Study it in advance, and bring it to the tutorial (don’t expect your tutor to give you one at the tutorial!). Solutions may be purchased at the end of each week from KOPY STOP (see above) or got from the Web (see below).

Assignments: Two, each marked out of 5. The due dates will be announced well in advance (lateness penalty normally 1 mark out of 5 per day or part thereof!). After each has been marked, solutions can be purchased at KOPY STOP (see above), or got from the Web (see below).

Consultations: Times for friendly lunch hour consultations in my room will be announced soon. You are most welcome at these. They are naturally most heavily frequented just before assignment due dates, but you should take advantage of slack times to clear up difficulties in understanding lectures and tutorial solutions.

The Web: Some material such as tutorial sheets and solutions will be put regularly on the Web, but you will need an acrobat reader (freely available!) to access it. Please visit http://www.maths.usyd.edu.au:8000/u/UG/SM/MATH3005

Assessment: 80% exam + 10% assignments + 10% tutorial participation.

Best wishes

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Here is the instruction list for a 5-state TM (i.e., 5 non-halting states), with a 
4-symbol alphabet \{b, A, B, X\} (hence \(4 \times 5 = 20\) instructions):

\[
\begin{align*}
q_1, & \ b, \ b, \ L, \ q_5; \\
q_1, & \ A, \ X, \ L, \ q_2; \\
q_1, & \ B, \ X, \ L, \ q_3; \\
q_1, & \ X, \ X, \ R, \ q_1; \\
q_2, & \ b, \ A, \ R, \ q_4; \\
q_2, & \ A, \ A, \ L, \ q_2; \\
q_2, & \ B, \ B, \ L, \ q_2; \\
q_2, & \ X, \ X, \ L, \ q_2; \\
q_3, & \ b, \ B, \ R, \ q_4; \\
q_3, & \ A, \ A, \ L, \ q_3; \\
q_3, & \ B, \ B, \ L, \ q_3; \\
q_3, & \ X, \ X, \ L, \ q_3; \\
q_4, & \ b, \ b, \ R, \ q_4; \\
q_4, & \ A, \ A, \ R, \ q_4; \\
q_4, & \ B, \ B, \ R, \ q_4; \\
q_4, & \ X, \ X, \ R, \ q_1; \\
q_5, & \ b, \ b, \ C, \ q_0; \\
q_5, & \ A, \ A, \ C, \ q_0; \\
q_5, & \ B, \ B, \ C, \ q_0; \\
q_5, & \ X, \ b, \ L, \ q_5.
\end{align*}
\]

What does this TM do if started on a tape of the form

\[
\cdots \ b \ b \ b \ * \ * \ \cdots \ * \ * \ b \ b \ \cdots
\]

where \(* \cdots *\) is a string, each term * of which is either an A or a B, and 
all other squares are blank, and the initially scanned square is the first * (shown 
underlined)?

Eg, if the initial tape is

\[
\cdots \ b \ b \ b \ A \ B \ B \ A \ B \ b \ b \ \cdots
\]

does the TM halt, and if so what is the halting tape?