Geometry of solitons

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Water waves in a canal, and pulses in an optical fibre, are both described by non-linear partial differential equations of the kind called 'completely integrable'. In either case, the general solution of the equation consists of a number of 'solitary waves' – or 'solitons' – which travel along without changing their shape, superimposed, in a non-linear way, upon a background of dispersing waves. The pure solitons are parametrized very explicitly by simple algebraic varieties. The purely dispersive solutions can be described by a kind of non-linear Fourier transform called the 'inverse-scattering' transform. They form an infinite-dimensional manifold which again has a simple geometrical description. It turns out that both kinds of solution are closely related to the group of loops in SU_2. The main aim of my talk is to explain how the two kinds of solution fit together to give the general solution of the equation.