The University of Sydney
Math1003 Integral Calculus and Modelling

Semester 2 **Exercises for Week 2** 2014

Assumed Knowledge: Sigma notation for sums. The ideas of a sequence of numbers and of the limit of a sequence. Sketching a curve given a function $f(x)$ or a set of tabulated values of a function $f(x)$.

Objectives:

(1a) To know what is meant by a Riemann sum.

(1b) To understand the definition of the definite integral of a function as the limit of a Riemann sum.

(1c) To understand how to interpret a definite integral of a function in terms of area when the function is not always positive.

(1d) To be able to use the upper and lower sums to obtain an estimate of the error in evaluating a definite integral.

(1e) To be able to find the number of subintervals (sampling frequency) required to reduce the error to a given value in the case of monotonic functions.

Preparatory questions:

1. (i) Evaluate the sum $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k}{k+2}$.

(ii) Write the following sum using sigma notation: $1 + \dfrac{x}{2} + \dfrac{x^2}{3} + \dfrac{x^3}{4} + \cdots + \dfrac{x^n}{n+1}$.

2. If $c(t)$ represents the cost, in dollars per day, to heat your house, where t is measured in days and $t = 0$ on 1 June 2005, what does $\int_0^{90} c(t)\, dt$ represent?

3. In a chemical reaction, the rate at which a precipitate is formed is a decreasing function of time. In an experiment the following rates were recorded.

time (s)	0	1	2	3	4	5	6
rate (g/s)	12	8.4	5.9	4.1	2.9	2.0	1.4

Sketch the rate as a function of time.

4. Sketch the curve $y = \sqrt{1 + x^3}$.

Practice Questions:

5. Using the data in Question 3,

(i) Find an over-estimate, and an under-estimate, for the total mass of precipitate formed in these six seconds.

(ii) How often would measurements have to be made in order to find under-estimates and over-estimates which differ by less than 1g from the exact mass of the precipitate formed in these 6 seconds?

6. Use upper and lower Riemann sums with 5 subintervals to estimate $\int_1^2 \sqrt{1+x^3}\,dx$.

7. Draw diagrams illustrating the approximation of $\ln 2 = \int_1^2 \frac{dt}{t}$ using upper and lower Riemann sums with 10 subdivisions.
Estimate $\ln 2$ correct to one decimal place by calculating the upper and lower Riemann sums.

More Exercises:

8. A speed-boat accelerates from rest (with increasing velocity), reaching a speed of 40 m/sec as it moves in a straight line over a period of 20 seconds. Velocities were measured every 2 seconds, and recorded in the following table:

time (sec)	0	2	4	6	8	10	12	14	16	18	20
vel (m/sec)	0	3	8	13	19	25	29	33	36	38	40

(i) Use lower and upper Riemann sums to find lower and upper bounds for the distance travelled by the boat.

(ii) How often would measurements need to be taken to guarantee that lower and upper Riemann sums differ from the actual distance travelled by less than 10 m?

Answers to Preparatory Questions

1. (i) $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k}{k+2} = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} = \frac{23}{60}.$

(ii) $\displaystyle 1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \cdots + \frac{x^n}{n+1} = \sum_{k=0}^{n} \frac{x^k}{k+1}.$

2. $\int_0^{90} c(t)\,dt$ represents the heating cost, in dollars, for 90 days from 1 June 2005, or roughly for the months June, July, and August.