THE UNIVERSITY OF SYDNEY
MATH1003 INTEGRAL CALCULUS AND MODELLING

Semester 2 | **Exercises for Week 5** | 2014

**Assumed Knowledge:** Sketching curves of simple functions. Integrals of simple functions such as $x^n$ (including $1/x$), $\sin x$, $\cos x$, $e^x$.

**Objectives:**

**(4a)** To understand and be able to use integration by parts to evaluate definite integrals.

**(4b)** To understand that an indefinite integral is a function.

**(4c)** To understand that differentiation and (indefinite) integration are inverse processes when applied to functions.

**(4d)** To be able to sketch a function given its derivative.

**(4e)** To be able to derive a reduction formula for an integral.

**Preparatory questions:**

**1.** Find the indefinite integrals

*(i)* $\displaystyle\int \tan x\,dx$. Hint: Write $\tan x = \sin x/\cos x$ and choose a suitable substitution.

*(ii)* $\displaystyle\int x^2 e^x\,dx$.

**Practice Questions:**

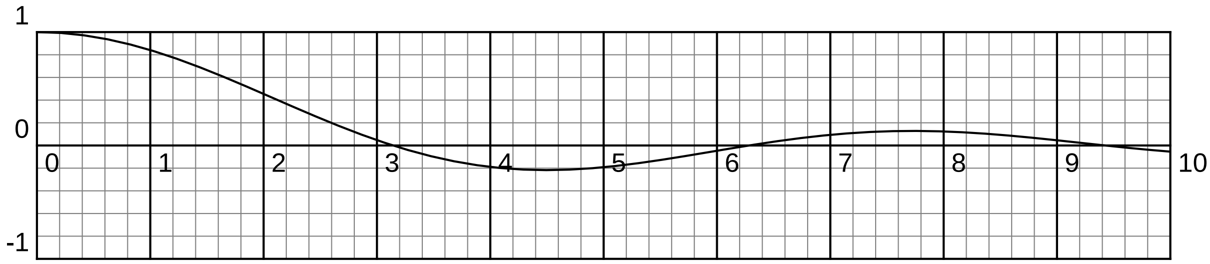
**2.** Evaluate the following integrals by using integration by parts.

*(i)* $\displaystyle\int_0^{1/2} xe^{2x}\,dx$. *(ii)* $\displaystyle\int_0^{\pi/4} \theta\sin 4\theta\,d\theta$. *(iii)* $\displaystyle\int_1^2 t^2\ln t\,dt$.

**3.** Define $\mathrm{Si}(x)$ as $\mathrm{Si}(x) = \displaystyle\int_0^x f(t)\,dt$, where $f(t) = \begin{cases} \dfrac{\sin t}{t} & t \neq 0 \\ 1 & t = 0\,. \end{cases}$

This function is called the *sine-integral*, and is useful in optics.

This is the graph of $f(t)$.




($i$) What is Si $'(x)$?.  ($ii$) What is the value of Si$(0)$?

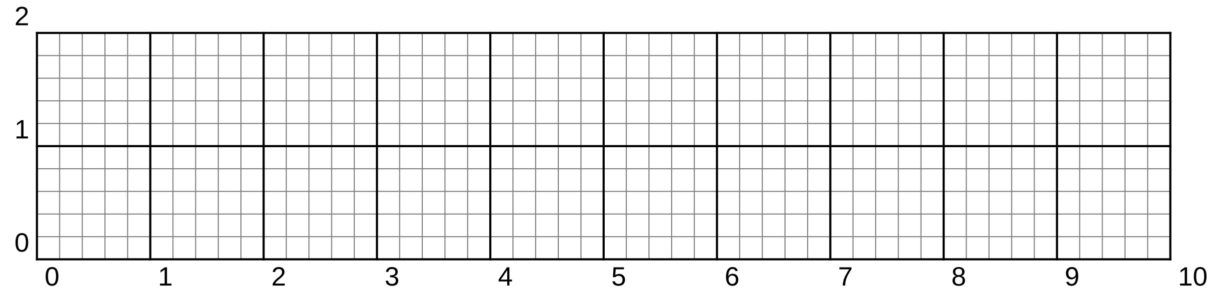
($iii$) For $0 \leq x \leq 3\pi$, use the graph of $f(t)$ to determine the values of $x$ for which Si$(x)$ is increasing, and the values of $x$ for which it is decreasing.

($iv$) For which values of $x$ between 0 and $3\pi$ does Si$(x)$ have stationary points?

($v$) Use the graph of $f(t)$ to estimate Si$(\pi)$, Si$(2\pi)$ and Si$(3\pi)$.

($vi$) Use the graph of $f(t)$ to determine values of $x$ at which Si has points of inflection.

($vii$) Sketch the graph of Si for $0 \leq x \leq 3\pi$.



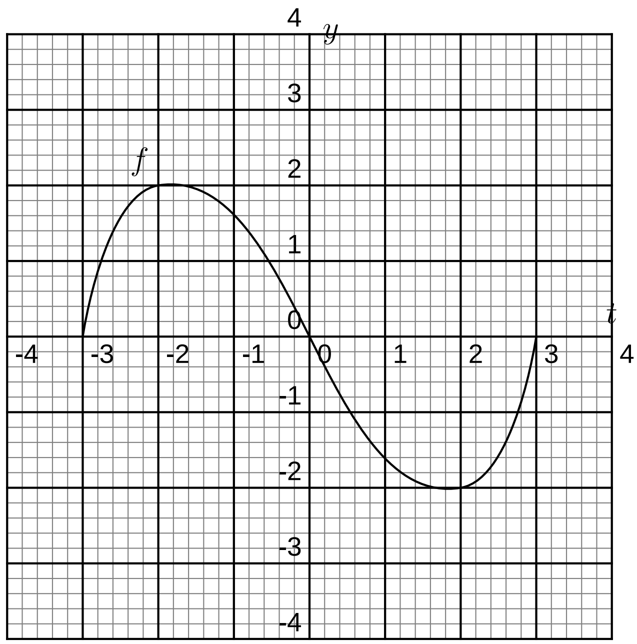

**4.** Establish the following reduction formula. [Hint: Write the integrand as $u(x)v'(x)$ where $u = \cos^{n-1} x$.]

$$\int \cos^n x\, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x\, dx\,.$$

Use this formula to find $\int \cos^2 x\, dx$ and $\int \cos^4 x\, dx$.

## More Exercises

**5.** Let $g(x) = \int_{-3}^{x} f(t)\, dt$ where $f$ is the *odd* function whose graph is shown.




$(i)$ Evaluate $g(-3)$ and $g(3)$.

$(ii)$ Estimate $g(-2)$, $g(-1)$ and $g(0)$.

$(iii)$ On what interval is $g$ increasing?

$(iv)$ Where does $g$ have a maximum value?

$(v)$ Sketch a rough graph of $g$.

**6.** Evaluate the following integrals by using integration by parts.

$(i)$ $\displaystyle\int_0^1 (2x+3)e^x\,dx.$ $\qquad$ $(ii)$ $\displaystyle\int_0^\pi \theta^2\cos 3\theta\,d\theta.$ $\qquad$ $(iii)$ $\displaystyle\int_{-\pi/4}^{\pi/4} t\sin t\cos t\,dt.$

Hint: First use an identity in $(iii)$.

**7.** Let $I_n = \int x^n e^x dx$. Use integration by parts to establish the reduction formula

$$I_n = x^n e^x - nI_{n-1}\,.$$

Hence find $\int x^3 e^x dx$.

**8.** $(i)$ Let $I_n = \int x(\ln x)^n dx$. Use integration by parts to establish the reduction formula

$$I_n = \frac{1}{2}x^2(\ln x)^n - \frac{n}{2}I_{n-1}\,.$$

$(ii)$ Starting with $I_0 = \int x\,dx = \frac{1}{2}x^2 + C$, use the reduction formula from part $(i)$ to find $I_2$.

## Answers to Preparatory Questions

**1.** $(i)$ $-\ln|\cos x| + C.$ $\qquad$ $(ii)$ $(x^2 - 2x + 2)e^x + C.$