THE UNIVERSITY OF SYDNEY
MATH1003 INTEGRAL CALCULUS AND MODELLING

Semester 2 **Practice Questions for Quiz 1** 2015

Quiz 1 will be held during your tutorial in week 7 (beginning 7 September).

The quiz questions will be based on material covered during lectures in weeks 1–5, which corresponds to material covered in tutorials in weeks 2–6.

The quiz will run for 40 minutes. You may use a non-programmable calculator. No other materials are permitted.

1. **a)** Find upper and lower Riemann sums for $f(x) = 2x^2 - 1$ on $[1, 6]$, using 5 equal subintervals.

b) What is the minimum number of equal subintervals required to compute a pair of upper and lower Riemann sums which differ by less than 0.001?

2. Find the following:

a) $\int \left(x\sqrt{x} + x^3\right) dx$; **b)** $\int_0^{\frac{\pi}{2}} \cos x \sin 2x \, dx$;

c) $\int_0^1 xe^{6x} \, dx$; **d)** $\int x\sqrt{x^2+1} \, dx$;

e) $\int_0^2 x\sqrt{x+1} \, dx$; **f)** $\int \cos^5 x \, dx$;

g) $\int_0^1 3^x \, dx$; **h)** $\int \sqrt{3}x(2-4x)^{1/3} \, dx$.

3. Find $\dfrac{dy}{dx}$ for $y = \dfrac{3^x \, x^{\sqrt{x}}}{\sqrt[3]{x^4 + 2x}}$.

4. Given that $\int_0^2 3f(x) \, dx = 6$ and $\int_6^0 f(x) \, dx = 7$, determine $\int_2^6 3f(x) \, dx$.

5. **a)** Find the area enclosed by the curves $y = \sqrt{x}$ and $y = x^4$.

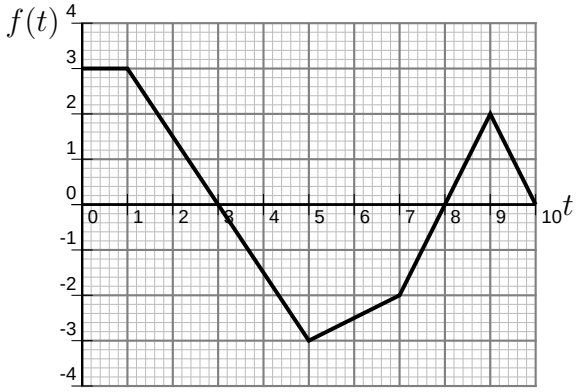
b) Find the volume of the solid formed when the area in part **a**) is rotated about

(*i*) the x-axis;

(*ii*) the y-axis;

(*iii*) the line $x = 2$.

6. Let $F(x) = \int_0^x f(t)\,dt$, for $0 \le x \le 10$, where f is the function whose graph is shown below.



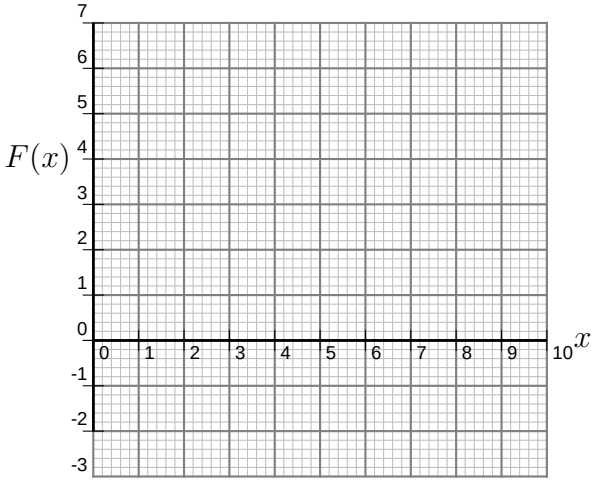
a) Determine the numeric values of $F'(5)$ and of $F''(8.5)$.

b) Compute $F(7)$.

c) For which values of x is the function $F(x)$ increasing?

d) Determine the positions of the local minima and local maxima of the function $F(x)$. Give your answer in the form (x, y).

e) Plot the function $F(x)$ on the graph below.

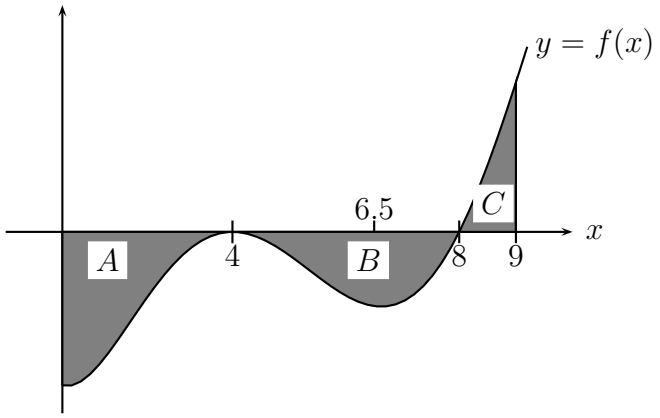


7. **a)** Find a reduction formula for $I_n = \int x^n \sin x\,dx$.

b) Hence calculate I_2, I_3.

8. Find the difference between the upper and lower Riemann sums for $f(x) = x^2$ on $[-3, 3]$ using 12 equal subintervals.

9. A, B and C represent the graphical areas between the curve $y = f(x)$ and the x-axis in the following diagram. Let $F(x) = \int_0^x f(t)\,dt$ for $0 \leq x \leq 9$.



Determine the following.

a) $F(9)$;

b) The values of x for which F is increasing;

c) The value of x at which F is a minimum.

d) The values of x at which $F''(x) = 0$.

ANSWERS

1. a) $L = 105$, $U = 175$. b) 350,001

2. a) $\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^4 + C$; b) $\frac{2}{3}$; c) $\frac{5e^6+1}{36}$; d) $\frac{(x^2+1)^{\frac{3}{2}}}{3} + C$;

e) $\frac{4(6\sqrt{3}+1)}{15}$; f) $\sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$; g) $\frac{2}{\ln 3}$;

h) $-\frac{3\sqrt{3}}{16}x(2-4x)^{4/3} - \frac{9}{448}\sqrt{3}(2-4x)^{7/3}$.

3. $\frac{3^x\, x^{\sqrt{x}}}{\sqrt[3]{x^4+2x}}\left(\ln 3 + \frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}} - \frac{4x^3+2}{3(x^4+2x)}\right)$

4. -27.

5. a) $\frac{7}{15}$;

b) i) $\frac{7\pi}{18}$; ii) $\frac{7\pi}{15}$; iii) $\frac{7\pi}{5}$.

6. a) -3 and 2; b) -2; c) $0 \le x < 3$ and $8 < x < 10$;

d) local minimum: $(8, -3)$, local maximum $(3, 6)$.

7. a) $I_n = -x^n \cos x + nx^{n-1}\sin x - n(n-1)I_{n-2}$;

b) $I_2 = 2x\sin x - (x^2-2)\cos x + C$; $I_3 = 3(x^2-2)\sin x - x(x^2-6)\cos x + C$.

8. 9

9. a) -A - B + C

b) $x \in (8, 9]$

c) 8 d) 4, 6.5.