How to read a mathematics textbook

R. Marangell[1] & D. Smith[2]

Expectations

- Reading mathematics is a very different process from reading other subjects, so your established strategies will need to be changed.
- Reading mathematics takes much longer than reading other technical subjects, and *vastly* longer than reading a novel

Purpose of the textbook

- Textbooks are written for students, to help them learn a topic in mathematics. Other kinds of mathematics books exist, and need reading in different ways, but these notes focus on reading mathematics textbooks.
- Usually, textbooks profess to be self-contained, with the implication that you should not need to read very much (or anything) outside the book to learn the subject. This is almost always completely false. Pretending that you only need one textbook for all of your vector calculus needs is like pretending that you only need one type of restaurant for any one type of cuisine. Take Italian food as an example. It would be quite rare indeed if you found a single restaurant anywhere in the world which had all types of Italian food (Sicilian, Milanese, Venetian, Tuscan, etc) and did it better than anywhere else. In fact, this is impossible, because my Aunt Louise doesn't cook in in a restaurant.
- So why do authors pretend that this is the case with the subject on which they're writing a book? I don't know.
  - It is important to get rid of this idea that the assigned book should be the end-all-be-all on the subject at hand.
  - It is also important to get rid of the idea that only one book will be sufficient to learn any given subject.
  - Your instructor probably plans a course from a wide variety of sources, so why shouldn't you also learn from such.
- Text books are chosen, hopefully by the instructor, for a variety of reasons. Big factors in the selection process include:
  - style,
  - order of material covered, and
  - the book they learned the material best from.
- Since these are entirely subjective, chances are high that you will find another textbook that will better suit you personally.

[1]University of Sydney, Australia
[2]Yale-NUS College, Singapore

- Other places to find text books on the same subject that you might prefer:
  - the library,
  - the American Institute of Mathematics free text book initiative,
  - personal websites of authors,
  - asking your instructor for other recommendations,
  - Googling ("free maths books online" returned $> 100,000,000$ results for example).
- You don't need to be constrained to just books.
  - Use online course notes.
  - Use Kahn academy.
  - Use stackexchange.
  - Use all of these things.
- Textbooks are chosen, ultimately, because instructors feel that they are the "best" choice for the material and course at hand.
- They should be reasonably comprehensive, and serve to unify notation.
  - Notation plays a big role in advanced mathematics, and getting everyone on the same page in this regard is very important.
  - Textbooks are a good way to get everyone using the same symbols to mean the same thing.
- Even if you disagree with your instructor's choice of primary textbook, and you've found what you think is a better one, it behoves you to read their book. These notes will hopefully help with that. They will also hopefully help you learn to read your favourite book.
- Textbooks are not encyclopedic. Clearly, you will be expected to have a firm grasp of some lower level mathematics before you begin reading. Look in the Preface / Introduction / Chapter 0 / Chapter 1 for a statement about what you should already know. Note that the Preface is often written for instructors and can be difficult for a student to understand.
- Textbooks often contain some (even a lot!) of information not required for your particular course.The reason there is so much extra information in the book is that the same textbook will be used for many similar courses at other universities. A topic that is important to your class might be considered ancillary in another course, and vice versa.

When to read the textbook

- This can vary between courses, so if your instructor says something different, then listen to them!
- Read the relevant (sub)section(s) before the class in which they are discussed.
- Do not attempt to read the whole book before the first day of class. This might seem like a good strategy—to get an overview of the material before the class begins—but in practice there are far too many deep, difficult concepts in a mathematics textbook for the strategy to be practical. If you try this, you will probably not actually have gained a clear overview, and will have wasted your time.
- Overviews can be useful. You might find one in the Preface / Introduction / Chapter 0 / Chapter 1, or the course syllabus. But if there isn't an overview to be found, then don't worry. Making an overview is a great way to revise for an examination.

- Read ahead—a little. It is rarely very helpful to have read §4.3 before §4.1 is discussed in class, and almost never valuable to have read §4.3 before §2.1 is covered in class (see above).
- The reason to read ahead a little is that it reduces the burden of catching up if you have a very demanding project in another class, or you get sick. Catching up if you miss a class is difficult, but reading mathematics when you are sick is also tough.

Steps to effective reading

Educational pedagogy suggests that I should come up with a snappy acronym or mnemonic to help you remember these steps to effectively reading a maths text. However my friend Dave wrote the steps, and I liked them, and so the acronym that we end up with is PRARSA, which while not exactly rolling off the tongue, does the job. If anyone has better ideas or acronyms/mnemonics, I am happy to hear them. In any case, you should:

1. Plan to read
2. Read for important concepts
3. Ask yourself questions
4. Read actively
5. Summarise
6. After a couple of days, reread

1 Plan to read

- Reading mathematics effectively takes a long time.
- Therefore it is important to set aside significant time to read.
- It is impossible to say how long to budget per page, as textbooks vary, but expect the order of 20–60 *minutes* per page.
  - Compare with a paperback novel, where it takes 30–90 *seconds* per page.
  - Don't expect to be able to read effectively:
    - on the bus,
    - with other people, even if they are reading the same section,
    - in 10-minute chunks of time, spread throughout the day,
    - while watching TV (although some people find silence difficult too; music you know well can be a good background),
    - while intoxicated.
  - If you don't plan to read, then you are less likely to do it, so plan when to read and record your plan in your schedule.

2 Read for important concepts

- This is the most difficult step, primarily because it is so unlike any kind of reading you have done before. With practice, you will get much quicker and much more confident.
- Aim to get a basic idea of what is important.
- Only read 1 section or subsection, no more than 10 pages.
- Focus on introductory paragraph(s).
  - You may need to look back at the introduction to §4.3 or even chapter 4 to get an overview of §4.3.5.
- Focus on concluding paragraph(s).
  - Occasionally, the introduction to the next (sub)section can help frame what was covered in the (sub)section you are reading.
- Focus on definitions.
  - Sometimes the value of a definition is immediately clear from its statement, or the surrounding text.
  - Often, objects satisfying recent definitions show up in theorems, and this can clarify why a definition matters.
  - Decide whether you understand the importance of the definition. "No" is an acceptable answer!
- Focus on theorems (numbered results).
  - The power of a theorem is often explained in the surrounding text.
  - The utility of a theorem can be demonstrated by worked examples before / after the theorem.
  - Decide whether you understand the importance of the theorem. "No" is an acceptable answer.
- A note on types of theorems:
  - Basically, *theorem* is the most important, followed by *proposition*, then *lemma*, and a *sublemma* is the least important.
  - Theorem means big, general (often abstract) result.
  - A proposition is either like a theorem, but a special case that might be easier to understand, or it is a major component of the very long and complicated proof of a more important theorem.
  - A lemma is a technical statement, with limited use on its own, but useful in the proof of a theorem or proposition.
  - A sublemma is used in the proof of a lemma, and is very unlikely to be used again outside the proof of that lemma.
  - This taxonomy is often abused. This usually happens for historical reasons; an original author might have called a result a "lemma", but it was realized decades later that the "lemma" was actually very important.
- Decide which are the most important theorems.
  - If a section contains 2 "theorems" and 5 "lemmas", then you can safely assume that the lemmas are less important on this first reading.
  - If a section contains only 1 "lemma" and no other numbered results, then the lemma is the important thing.

  - Any theorem (or proposition or lemma) that has a person's name attached to it is certainly very important to mathematics in general, but not necessarily important to the topic you are studying. It might just be a powerful tool from another area of mathematics, brought in to make the proof of theorem 2 work, and it is theorem 2 that is really important for the topic at hand.

- Focus on the worked examples.
  - Do you understand the problem being solved in each worked example? "No" is an acceptable answer!
  - You should not try to actually to follow the argument yet.
- Focus on the exercises at the end of the (sub)section.
  - Do you understand the problem posed in each exercise? "No" is an acceptable answer!
  - You should not try to actually solve the problem yet.
- Do not focus on:
  - proofs,
  - derivations,
  - detailed arguments in worked examples,
  - solving exercises.
- Read without a highlighter.
  - After step 4 (Read actively), your idea of what is important may change, so now you might be highlighting the wrong parts.
  - Highlighting / annotating the wrong parts of the text will reduce its readability for you in the future.
  - Highlighting / annotating the textbook will reduce its resale value.
- Read with a pen and paper.
  - Note down the things you have been focusing on, your thoughts about them, and what you do not understand.
  - I find it is helpful to pretend to be writing a lecture. I write things that I think will help clarify my imaginary audience.
- There is no need to read in order.
  - Try jumping around: introduction, conclusion, theorem 1, theorem 2, then the definitions they rely upon, then explanations of each.
  - Find a style that suits you, but keep experimenting occasionally.
  - I like to read from "inside out". Once I have identified the important theorems, I read what is before and after them, and then what is before and after that, and so on, until I hit the beginning of the section/chapter.

3 Ask yourself questions

- Based on your notes from step 2, decide what you do not understand.
- This will be a lot, but focus on the big things:
  - What does definition 1 mean?
  - How does exercise 4 relate to the theorems of this (sub)section?
  - Theorem 2 seems like it is obviously true, so why does its proof (which I have not yet even tried to read) cover 2 pages?
- Write down your questions.

4 Read actively

- This is the most time-consuming part, but you will get quicker with practice.
- Read with a pen and paper.
  - Take detailed notes.
  - Expand on every statement that is not quickly clear to you.
  - Complete every part of an argument that is "left as an exercise for the reader" or "very similar to the above proof". This is often annoying. Do it anyway.
- Read proofs, derivations, and worked examples carefully.
  - Check you understand every step.
  - Regularly look back at the statement of the theorem, and any relevant definitions, to keep in mind where the proof is aiming.
- Read with a pen and paper.
  - Try to anticipate questions that other people might have with the material.
  - How would you answer them?
- Interrogate statements of theorems in relation to their proofs.
  - A typical theorem takes the form "Suppose A, B, and C. Then D."
  - Make sure you understand where each of A, B, and C are used in the proof.
  - If you do not see where B is used, bear in mind that it might be the criterion of a lemma, or of a theorem from an earlier section, and that that lemma / theorem might be being used implicitly.
- Read with a pen and paper.
- Interrogate definitions in relation to their applications.
  - The precise phrasing of a definition is important in mathematics. Seeing how the definition is used in a subsequent proof can help to clarify why the definition was phrased in that particular way.
- Read with a pen and paper.
- Complete the exercises as best you can.

- If your instructor does not tell you which exercises to do, it is safe to assume they expect you to do (or at least attempt) attempt all of them.
- Difficulty of exercises varies greatly between textbooks, and even between the first and last exercises of the section, so do not worry if you can't do all of them. Though unusual, it is not unheard-of for upper-division textbooks to contain "exercises" that even the author does not know how to solve.
- If your instructor has told you not to attempt certain exercises, then it is probably not a good use of your time to try them anyway. The exercise might be much too difficult, outside the scope of your course, or even (rarely!) wrong. It is better to ask than get frustrated and waste time.

- Read with a pen and paper.

5 Summarize

- This will not take you much time, but will be valuable to you later.
- Do not use a highlighter on the original text.
- You will now have many pages of notes from steps 2–4.
- Make sure you have answered all of your questions from step 3.
- Find a way to summarize the important features of the (sub)section.
  - Refer to definitions, theorems etc. by number.
  - Write a few sentences containing the important ideas and how they relate.
  - Or list the important ideas, and show how they relate with arrows.
  - If a section is devoted entirely to proving a particular theorem, use a mind map to show how the lemmas depend upon one another.

6 After a couple of days, reread

- Do not reread too soon, but do reread before the class.
  - Approximately 2 days is optimal.
  - You could read the next section in between.
- Read your own summary first, then read the text again, checking your notes.
- Try one or two exercises again, limiting how much you look at your notes and the text, but do look if you get stuck.
- Try again at any parts you could not solve before.

General advice

- Use the index, index of symbols, contents page, and appendices.
  - This can save a lot of time in finding definitions you covered earlier but forgot.
- If your instructor gave you an erratum for the textbook, then use it! If not, look up the book online. Often errata are published for free.

- It is OK to not understand. In fact, it is necessary
  - If you are stuck on a particular point, make a note of it and come back.
  - It may become clear later on when you read the next page.
  - It may be a typographical error in the book. There are always some in any book. Because the language of mathematics is so concise, it can be very difficult for a non-expert to spot an error.
- Ask other students.
  - Although it is a bad idea to read together, if you can't understand a particular point, it can be helpful to email a friend, or post a question to a class web forum.
- Ask the instructor
  - Bring your notes to class and ask the instructor about the points you did not understand.
  - Your instructor might be available to answer questions in office hours, by email, before, or after class. Check the syllabus or ask to find out the policy on this.
- Read the figures with the text.
  - Diagrams and text explain one another. Keep looking back and forth between the two.
- Take breaks.
  - Be active in breaks. Even walking to the other side of the room and back can help.
  - The level of concentration required to read mathematics effectively is very high. When your concentration starts to weaken, take a short break, rather than wasting time by half-concentrating.
- Reading mathematics effectively is difficult but, with tenacity and practice, you really can do it!
- You will get much quicker, more confident, and more accurate with practice.