I've been asked to talk about my experiences in school teaching and syllabuses, so I ask you for the next few minutes to go back to a time before you came to this august place.

A school is a wonderful place. The liveliness and energy of children is astonishing. Physically, they never stop moving. Morally, they test every boundary you set. Intellectually, everything is new and exciting, and they want to question things to the limit. A school is trying to give leadership in these three areas, in company with the parents, as a child seeks maturity. It is not a dull job.

A school classroom comes alive when the freshness of the children's imagination encounters the knowledge and rhetoric of a well-educated teacher. I can assure you that strong knowledge of mathematics, including post-graduate or doctoral studies, is extremely valuable in producing the extraordinary energy that characterises a successful classroom. I warn you, do not discuss with a Year 7 class in full flight whether $0{\cdot}\dot{9}$ is 1, or whether there are infinitely many whole numbers, or whether -5 is a prime, if you are not secure in least upper bounds, contrasting interpretations of infinity, and Euclidean domains, or they'll run rings round you.

I first started teaching fractions to Year 7 as is done in most textbooks, illustrating the topic with little circles divided up into equal sectors, some shaded, some not. So to show that $\frac{2}{3} = \frac{4}{6}$, you have six sectors with four of them shaded. These things worked reasonably, but they weren't as convincing as I had expected, and I was uneasy about them. I thought about Max Kelly's lectures on the quotient construction of the rationals from the integers, where the equality of equivalent fractions is actually part of the definition of a fraction. I looked at the textbook's diagram of the number line, with $\frac{2}{3}$ marked as a number between 0 and 1. Of course they have to know that, but I remembered TG Room's and Phil Kirkpatrick's lectures on projective geometry, with their homogeneous coordinates — maybe these fractions are points on a projective line. Projective geometry, by the way, saves the day countless times in school mathematics.

And then you realise what's happening — the analysts have got at this topic, and sucked out the algebra and the geometry. Placing $\frac{2}{3}$ between 0 and 1 is normalisation, and ratios get lost in normalisation. Then you look again at the areas involved in the circles and the sectors — this is integration, not arithmetic! This is Euclid's interpretation of numbers as lengths and areas, which is why cuisenaire rods failed. Of course they need to know these things, but not right now. Instead, you pick up Pythagoras' much earlier interpretation of numbers as rectangular arrays, and teach fractions as operators on those arrays. There's a block of twelve points — three rows of four — and two thirds means that you take two of the three rows, which is eight points. Or you arrange the twelve points as six rows of two, and take four of the six rows — still eight points. That's what $\frac{4}{6} = \frac{2}{3}$ starts off meaning — it's the equality of two operators. Everything is an operator, just as Birkhoff and Maclane teach. In no time flat, you have arrays all over the board. You are soon multiplying fractions and cancelling them, and the whole machinery of HCFs and LCMs and primes becomes immediately visible in the arrays, so that fractions are now seamlessly integrated with all the previous whole number arithmetic and ratio work right at the start of the topic.

Of course you go back later and do ratios of lengths, and of areas, and place $\frac{2}{3}$ where it belongs on the number line. But at the start, you expound things in terms of what you decide are the simplest ideas. You certainly do not chatter about projective points, or quotient rings, or operators, or integration — that would ruin everything. And by the way, next year you see some different unity that takes you off on quite a different path. I assure you that a similar account applies to every detail throughout high school, and every time you use your knowledge to give a simpler and clearer account, that essential liveliness appears spontaneously in the classroom, because children take an extraordinary delight in clarity of thought.

There is another reason for people with strong academic backgrounds to go into school teaching. You can't produce a lively and effective classroom without syllabuses that are firmly based in the language and structure of the discipline, and syllabus development desperately needs people who combine experience of the classroom with strong knowledge of their discipline. The last twenty five years have been most unfortunate. The universities have largely failed in their most basic duty to the community, giving up their previous role as guardians of their disciplines, and handing over stewardship to the non-discipline of education and to government bureaucracies. Physics is now taught with little mathematics, with much basic physics ignored. Chemistry is muddled by an organisation based on relevance instead of the structures of chemistry. Parts of English have been confused by concerns of sociology and cultural studies. The well-intentioned desire to teach children about important social issues, like anti-discrimination and global warming, has in some syllabuses disturbed traditional liberal principles of teaching the sound knowledge and reasoning that should underpin educated judgement about those very social issues.

So far, Mathematics in NSW has been a little bruised, but has remained largely intact, for three good reasons. First, children and their teachers love the courses, particularly their rigour and imagination, their unity and their sense of purpose. Secondly, each successive attempt to degrade the courses has been so riddled with mistakes and misunderstandings that it has been an easy target. But most importantly, there has been an at times ferocious defence of the syllabuses by teachers, and by academics who do understand very well the great responsibility they have to the community, and I want to thank Gus Lehrer particularly for his timely interventions in recent years. My previous Headmaster adds a fourth reason, "You mathematicians always get what you want, because no one knows what you are talking about". Nevertheless, we have had twenty five years of chaos, with the mathematics syllabuses coming under continuous attack from successive waves of misplaced enthusiasms — for so-called 'understanding' in place of algorithmic ability, for calculators and then for computers, for unworkable assessment procedures, for unbounded problem solving and groupwork, for 'gifted and talented' and acceleration, for various people's pet mathematical topics, for bureaucratic uniformity across disciplines, all of course under the slogans of 'relevance' and 'preparing children for the modern world'. Things can't go on like this with such dangerous instability, and I continue to appeal most strongly to the universities to take back their authority and stewardship over school syllabuses in all disciplines so that schools can do their job of preparing children for adulthood with whatever mixture of liberal and vocational education is appropriate for them. The National Curriculum is shaping up as a serious test of the universities' resolve.

But I return to the wonderful transforming world of the classroom. You want power? You have more power than a king. A few years ago, I gave a routine spreadsheet problem to a Year 11 class. Most of them couldn't do it, because they couldn't use the 'fill down' procedure. I threw a tanny, and told them that if they wanted to get on in life, they had to learn to drive a car, programme a spreadsheet, and play the piano. A few years later I found that a very able boy, who was also a violin virtuoso, had gone home that night and started learning the piano. Even the Prime Minister can't produce a stimulus package like that.

Occasionally, however, you'll perhaps be chatting to a boy in the class and find that not only does he know the Sylow theorems, but that he can prove them. Time to hand him over to Gus Lehrer, and Geoff Ball, and Terry Gagen, and the Olympiad and Summer School programmes. These academics are the capable hands in which you prize winners now find yourselves, and you are most fortunate to be in such an excellent place here in this Mathematics Department, with people who take such great delight in their mathematics and who are so generous in their desire to share their knowledge with their students. You are also fortunate to have the intellectual capacity to understand what they are talking about. I congratulate you on your achievements, and wish you every good fortune in your future careers.

Bill Pender, April 2009