On 4-dimensional mapping tori. II

Jonathan A. Hillman

School of Mathematics and Statistics, The University of Sydney,

Sydney, NSW 2006, AUSTRALIA

e-mail: jonh@maths.usyd.edu.au

ABSTRACT

We show that a PD_4-complex X is homotopy equivalent to the mapping torus of a self homotopy equivalence of a PD_3-complex if and only if $\chi(X)=0$ and $\pi_1(X)$ is an extension of Z by a finitely presentable normal subgroup.

AMS Subject Classification (1991): Primary 57P10. Secondary 57N13.

Key words and phrases: cup product. 4-manifold. mapping torus. Poincaré duality.

Let M be a closed n-manifold and $f : M \to S^1$ a map which induces an epimorphism f_* on fundamental groups. Stallings showed that if M is an irreducible aspherical 3-manifold then f is homotopic to a bundle projection if and only if f_* has finitely generated kernel, while Farrell showed that if $n \geq 6$ this is so if and only if the homotopy fibre of f is finitely dominated and a torsion invariant $\tau(f) \in Wh(\pi_1(M))$ is 0 [St62, Fa71]. The corresponding results in dimensions 4 and 5 are constrained by the present limitations of geometric topology in these dimensions. (In fact there are counter-examples to the most natural 4-dimensional analogue of Farrell's theorem [We87]).

There are however homotopy theoretic analogues which apply in all dimensions. Milnor showed that if the infinite cyclic covering space associated to the kernel of f_* (i.e., the homotopy fibre of f) has finitely generated homology then it satisfies Poincaré duality with simple coefficients of formal dimension $n-1$ [Mi68]. Barge extended this to covering spaces associated with other "weak duality groups" [Ba80,80']. Quinn and Gottlieb obtained a much stronger result, requiring more stringent hypotheses. Namely, the homotopy fibre of a map between Poincaré duality complexes is again a Poincaré duality complex if and only if it is finitely dominated [Go79].

We are interested in providing a 4-dimensional homotopy fibration theorem with hypotheses similar to those of Stallings and a conclusion similar to that of Quinn and Gottlieb. In [Hi98] we gave criteria for a PD_4-complex to be homotopy equivalent to the mapping torus of a self homotopy equivalence of a PD_3-complex which involved the Euler characteristic, the fundamental group and an injectivity condition on a certain homomorphism. Here we shall show that the latter condition always holds, and so the obvious simple necessary conditions on χ and π_1 are also sufficient. In §3 we decribe explicitly some of the isomorphisms arising in the special case when $\nu = 1$, and in §4 we consider coverings associated to other PD-groups.

As in our earlier papers on this topic, our argument is prompted by the work of Barge.

§1. Injectivity

We shall begin by summarizing the definitions and notation from [Hi89, Hi98].

Let π be an extension of Z by a normal subgroup ν and fix an element t of π whose image generates π/ν. Let $\alpha : \nu \to \nu$ be the automorphism determined by $\alpha(h) = tht^{-1}$ for all h in ν. This automorphism extends to a ring automorphism (also denoted by α) of the group ring $R = \mathbb{Z}[\nu]$, and the ring $S = \mathbb{Z}[\pi]$ is a twisted Laurent extension, $S = R_\alpha[t, t^{-1}]$. Let $\Lambda = \mathbb{Z}[t, t^{-1}]$.

If M is a left S-module let $M|_R$ denote the underlying R-module, and let $\hat{M} = Hom_R(M|_R, R)$. Then $\hat{M}$ is a right R-module via $(f\xi)(m) = f(m)\xi$ for all ξ in R, f in $\hat{M}$ and m in M. If $M = S$ then $\hat{S}$ is also a left S-module via $(\phi t^r f)(\xi t^s) = \xi\alpha^{-s}(\phi)f(t^{s-r})$ for all f in $\hat{S}$, ϕ, ξ in R and r, s in Z. As the left and right actions commute $\hat{S}$ is a (S, R)-bimodule. The homomorphism $j : \hat{S} \to R[[t, t^{-1}]] = \{\Sigma_{n\in Z} t^n\phi_n \mid \phi_n \in R\}$ given by $j(f) = \Sigma t^n f(t^n)$ for all f in $\hat{S}$ is a (S, R)-bimodule isomorphism.

For each f in $\hat{M}$ we may define a function $T_M f : M \to \hat{S}$ by the rule $(T_M f)(m)(t^n) = f(t^{-n}m)$ for all m in M and n in Z. Then $T_M f$ is S-linear, and $T_M : \hat{M} \to Hom_S(M, \hat{S})$ is an isomorphism of right R-modules. Thus we have a natural equivalence $T : Hom_R(-|_R, R) \Rightarrow Hom_S(-, \hat{S})$ of functors from $\mathbf{Mod}_S$ to $\mathbf{Mod}_R$. If C_* is a chain complex of left S-modules then T induces natural isomorphisms from $H^*(C_*|_R; R) = H^*(Hom_R(C_*|_R, R))$ to $H^*(C_*; \hat{S}) = H^*(Hom_S(C_*, \hat{S}))$. In particular, since the forgetful functor $-|_R$ is exact and takes projectives to projectives there are isomorphisms from $Ext^*_R(M|_R, R)$ to $Ext^*_S(M, \hat{S})$ which are functorial in M.

If M and N are left S-modules let $M \otimes N$ denote the tensor product over $\mathbb{Z}$ with the diagonal left π-action, defined by $g(m \otimes n) = gm \otimes gn$ for all $m \in M$, $n \in N$ and $g \in \pi$. The function $p_M : \Lambda \otimes M \to M$ defined by $p_M(\lambda \otimes m) = \lambda(1)m$ is then an S-linear epimorphism.

The additive function $e_S : \Lambda \otimes \hat{S} \to S$ defined by $e_S(t^n \otimes f) = t^n f(t^n)$ for all f in $\hat{S}$ and n in Z is S-linear. We may use e_S to define products in cohomology. Let A_* be a Λ-chain complex and B_* a S-chain complex and give the tensor product the total grading $A_* \otimes B_*$ and differential and the diagonal π-action. Let $e_{S\sharp}$ be the change of coefficients homomorphism induced by e_S, and let $u \in H^p(A_*;\Lambda)$ and $v \in H^q(B_*;\hat{S})$. Then $u \otimes v \mapsto e_{S\sharp}(u \times v)$ defines a pairing from $H^p(A_*;\Lambda) \otimes H^q(B_*;\hat{S})$ to $H^{p+q}(A_* \otimes B_*;S)$.

Now let A_* be the Λ-chain complex concentrated in degrees 0 and 1 with A_0 and A_1 free of rank 1, with bases $\{a_0\}$ and $\{a_1\}$, respectively, and with $\partial_1 : A_1 \to A_0$ given by $\partial_1(a_1) = (t-1)a_0$. Let $\eta_A : A_1 \to \Lambda$ be the isomorphism determined by $\eta_A(a_1) = 1$ and let $\alpha_A : A_0 \to \mathbb{Z}$ be the augmentation determined by $\alpha_A(a_0) = 1$. Then $[\eta_A]$ generates $H^1(A_*;\Lambda)$. Let B_* be a projective S-chain complex and let $p_{B*} : A_* \otimes B_* \to B_*$ be the chain homotopy equivalence defined by $p_{Bj}((\lambda a_0) \otimes b_j) = \lambda(1)b_j$ and $p_{Bj}((\lambda a_1) \otimes b_{j-1}) = 0$, for all $\lambda \in \Lambda$, $b_{j-1} \in B_{j-1}$ and $b_j \in B_j$. Let $j_{B*} : B_* \to A_* \otimes B_*$ be a chain homotopy inverse to p_{B*}. Let $h_S([\phi]) = j_B^* e_{S\sharp}([\eta_A] \times [\phi])$ for $\phi : B_q \to \hat{S}$ such that $\phi\partial_{q+1} = 0$. If $f : C_* \to B_*$ is a chain homomorphism of projective S-chain complexes then $h_S([\phi f_q]) = f^* h_S([\phi])$. Thus we obtain homomorphisms $h_S : H^q(B_*;\hat{S}) \to H^{q+1}(B_*;S)$ which are functorial in B_*. In particular, if B_* is a projective resolution of the S-module M we obtain homomorphisms $h_S : Ext^q_S(M,\hat{S}) \to Ext^{q+1}_S(M,S)$ which are functorial in M. Since cross product with a 1-cycle induces a degree 1 cochain homomorphism, these homomorphisms induce homomorphisms between universal coefficient spectral sequences.

Lemma 1. *Let M be an S-module such that $M|_R$ is finitely generated as an R-module. Then $h_S : Hom_S(M,\hat{S}) \to Ext^1_S(M,S)$ is injective.*

Proof. Let B_* be a projective resolution of the S-module M and let $q : B_0 \to M$ be the defining epimorphism (so that $q\partial_1 = 0$). We may use composition with q to identify $Hom_S(M,\hat{S})$ with the submodule of 0-cocycles in $Hom(B_*,\hat{S})$, and we set $h_S(\phi) = h_S([\phi q])$ for all $\phi : M \to \hat{S}$.

Let $\phi : M \to \hat{S}$ be a homomorphism and let $g = \phi q : B_0 \to \hat{S}$. We may write $g(b) = \Sigma t^n g_n(b) = \Sigma t^n g_0(t^{-n}b)$, where $g_0 : B_0 \to R$ is R-linear (and $g_0\partial_1 = 0$). If $h_S(\phi) = 0$ then $e_\sharp([\eta_A] \times [g]) = \delta f$ for some S-linear homomorphism $f : A_0 \otimes B_0 \to S$. Therefore $g_0(b) = f((t-1)a_0 \otimes b)$ for all $b \in B_0$, while $f(1 \otimes \partial_1) = 0$. Let $k(b) = f(a_0 \otimes b)$ for $b \in B_0$. Then $k : B_0 \to S$ is R-linear, and $k\partial_1 = 0$, so k factors through M. In particular, $k(B_0)$ is finitely generated as an R-submodule of S. But as $S = \bigoplus t^n R$ and $g_0(b) = tk(t^{-1}b) - k(b)$ for all $b \in B_0$, this is only possible if $k = g_0 = 0$. This proves the lemma. $\square$

§2. The main result

Suppose now that B_* is a projective S-chain complex such that $B_j = 0$ for $j < 0$, $H_0(B_*) \cong \mathbb{Z}$ and $H_1(B_*) = 0$. Then the quotient epimorphism from B_0 to $H_0(B_*)$ factors through Λ and we may choose a chain homomorphism $\epsilon_{B*} : B_* \to A_*$ which induces an isomorphism $H_0(B_*) \cong H_0(A_*)$. Let $\alpha_B = \alpha_A \epsilon_{B_0} : B_0 \to \mathbb{Z}$ and $\eta_B = \eta_A \epsilon_{B_1} : B_1 \to \Lambda$. Then α_B is a generator of $H^0(B_*; \mathbb{Z})$ and $[\eta_B] = \epsilon_B^*([\eta_A])$ is a generator of $H^1(B_*; \Lambda)$.

Lemma 2. *Let X be a connected space with $\pi_1(X) \cong \pi$ and let $B_* = C_*(\tilde{X})$ be the singular chain complex of the universal cover of X. Then $h_S([\phi]) = [\eta_B] \cup [\phi]$.*

Proof. The Alexander Whitney diagonal approximation $d_* : B_* \to B_* \otimes B_*$ is π-equivariant, if the tensor product is given the diagonal left π-action, and we may take $j_{B*} = (\epsilon_B \otimes 1)d_*$ as a chain homotopy inverse to p_{B*}. Therefore $h_S([\phi]) = d^* e_{S\sharp}([\eta_B] \times [\phi]) = [\eta_B] \cup [\phi]$. $\square$

Thus we may identify the homomorphisms h_S with the cup-products involved in our description of duality.

We are now able to prove the theorem.

Theorem 3. *A finite PD_4-complex M with fundamental group π is homotopy equivalent to the mapping torus of a self homotopy equivalence of a PD_3-complex if and only if $\chi(M) = 0$ and π is an extension of Z by a finitely presentable normal subgroup ν.*

Proof. By Theorem 4 of [Hi98] it is sufficient to observe that if $\Pi = H^2(\pi; \mathbb{Z}[\pi])$ then $\Pi|_\nu \cong H^1(\nu; \mathbb{Z}[\nu])$ is finitely generated as a $\mathbb{Z}[\nu]$-module, and so the map from $Hom_{\mathbb{Z}[\nu]}(\Pi|_\nu, \mathbb{Z}[\nu])$ to $Ext^1_{\mathbb{Z}[\pi]}(\Pi, \mathbb{Z}[\pi])$ induced by h_S is a monomorphism, by Lemma 1. $\square$

A rather different proof of this theorem could be given using Ranicki's criterion for an infinite cyclic cover to be finitely dominated [Ra95] and the Quinn-Gottlieb theorem, if finitely generated stably free modules of rank 0 over the Novikov rings $A_\pm = \mathbb{Z}[\nu]_\alpha((t^{\pm 1}))$ are trivial. (For $H_*(A_\pm \otimes_\pi C_*)$ is a stably free module of rank 0, concentrated in degree 2, by the argument of Lemma 2.3 of [Wl]. See also Chapter II of [Hi]).

The hypothesis that ν be almost finitely presentable (FP_2) suffices to show that M_ν satisfies Poincaré duality with local coefficients. Finite presentability is used only to show that M_ν is finitely dominated. In view of the fact that 3-manifold groups are coherent, we might hope that the condition on ν could be weakened still further to require only that it be finitely generated.

Current technology does not enable us to decide whether M_ν is homotopy equivalent to a closed 3-manifold. However we may use Farrell's theorem to obtain a partial reduction to the case of a cartesian product. The projection of π onto $\mathbb{Z}$ may be realized by a map $f : M \to S^1$ with homotopy fibre

M_ν. The composite of f with the projection of $M \times S^1 \times S^1$ onto M defines a map $g : M \times S^1 \times S^1 \to S^1$ with homotopy fibre $M_\nu \times S^1 \times S^1$ which is again finitely dominated (in fact, homotopy finite). The homotopy equivalence associated to g by Farrell is obtained from the one associated to f by product with $id_{S^1 \times S^1}$ and so has torsion $\tau(g) = \tau(f)\chi(S^1 \times S^1) = 0$ in $Wh(\pi \times Z^2)$. Therefore if M is a closed 4-manifold g is homotopic to a bundle projection and so $M_\nu \times S^1 \times S^1$ is homotopy equivalent to a closed 5-manifold.

There are however 4-manifolds which are simple homotopy equivalent to $S^1 \times RP^3$ (and thus satisfy the hypotheses of our theorem) but which are not homeomorphic to mapping tori [We87].

§3. The case $\nu = 1$

When $\nu = 1$ (so $R = \mathbb{Z}$ and $S = \Lambda$) we can relate h_Λ to other more explicit homomorphisms. Let M be a Λ-module which is finitely generated as an abelian group, and let N be its maximal finite submodule. Let λ_M be the minimal polynomial of t, considered as an automorphism of $(M/N)|_\mathbb{Z}$. Then $Ann_\Lambda(M/N) = (\lambda_M)$ [Hi81]. Let $j_\lambda : \Lambda/(\lambda) \to \mathbb{Q}(t)/\Lambda$ be the homomorphism sending the class of g to $\frac{g}{\lambda}$. Then j_{λ_M} induces an isomorphism $Hom_\Lambda(M, \Lambda/(\lambda_M)) \cong D(M) = Hom_\Lambda(M, \mathbb{Q}(t)/\Lambda)$. We shall show that $D(M)$ is isomorphic to each of $\hat{D}(M) = Hom_\Lambda(M, \hat{\Lambda})$, $E(M) = Ext^1_\Lambda(M, \Lambda)$ and $F(M) = Hom_\mathbb{Z}(M|_\mathbb{Z}, \mathbb{Z})$.

Since $M|_\mathbb{Z}$ is finitely generated the leading coefficient and constant term of λ_M are ± 1, and we may define a homomorphism $\rho_M : D(M) \to F(M)$ by $\rho_M(\phi)(m) = -Res(\phi(m)\frac{dt}{t}, 0) - Res(\phi(m)\frac{dt}{t}, \infty)$, where $Res(\omega, a)$ is the residue of the differential form ω at a. (Note that if $u = t^{-1}$ then $\frac{du}{u} = -\frac{dt}{t}$, and so $Res(f\frac{dt}{t}, \infty) = -Res(\bar{f}\frac{du}{u}, 0)$, where $\bar{f}(u) = f(u^{-1})$). Let

$T_M : F(M) \to \hat{D}(M)$ be the isomorphism defined as in [Ba80'] (and as above) by $T_M f(m) = \Sigma t^n f(t^{-n}m)$. Then $T_M \rho_M(\phi)(m) = Lexp(\phi(m), \infty) - Lexp(\phi(m), 0)$, where $Lexp(f, a)$ is the Laurent expansion of the rational function f about a, since $Lexp(f, 0) = \Sigma t^n Res(t^{-n} f \frac{dt}{t}, 0)$ and $Lexp(f, \infty) = -\Sigma t^n Res(t^{-n} f \frac{dt}{t}, \infty)$.

Let B_* be a projective resolution of M. If $\phi : M \to \mathbb{Q}(t)/\Lambda$ let $\phi_0 : B_0 \to \mathbb{Q}(t)$ be a lift of ϕ. Then $\phi_0 \partial_1$ has image in Λ, and so defines a homomorphism $\phi_1 : B_1 \to \Lambda$ such that $\phi_1 \partial_2 = 0$. Consideration of the short exact sequence of complexes $0 \to Hom_\Lambda(B_*, \Lambda) \to Hom_\Lambda(B_*, \mathbb{Q}(t)) \to Hom_\Lambda(B_*, \mathbb{Q}(t)/\Lambda) \to 0$ shows that $\delta(\phi) = [\phi_1]$, where $\delta : D(M) \to E(M)$ is the Bockstein homomorphism associated to the coefficient sequence. (The extension corresponding to $\delta\phi$ under the isomorphism of Theorem III.2.4 of [HS] is the pullback over ϕ of the sequence $0 \to \Lambda \to \mathbb{Q}(t) \to \mathbb{Q}(t)/\Lambda \to 0$).

Lemma 4. *The natural transformations h_Λ and ρ are equivalences, and $h_\Lambda T \rho = \delta$.*

Proof. As every homomorphism from M to $\mathbb{Q}(t)/\Lambda$ factors through $\Lambda/(\lambda_M)$ it suffices to check that $h_\Lambda T_M \rho_M(j_\lambda) = \delta(j_\lambda)$ for $\lambda = \lambda_M$. Moreover we may extend coefficients to $\mathbb{C}$, and so we may reduce to the case $\lambda = (t - \alpha)^n$.

We may assume that B_1 and B_0 are freely generated by b_1 and b_0, respectively, and that $\partial(b_1) = \lambda b_0$. The chain homotopy equivalence j_{B*} may be defined by $j_0(b_0) = a_0 \otimes b_0$ and $j_1(b_1) = a_0 \otimes b_1 + \Sigma\beta_{pq}(t^p a_1) \otimes (t^q b_0)$, where $\Sigma\beta_{pq}x^p y^q = (\lambda(xy) - \lambda(y))/(x-1) = y\Sigma_{0 \le r < n}(xy - \alpha)^r (y - \alpha)^{n-r-1}$. (This formula arises naturally if we identify $\Lambda \otimes_{\mathbb{Z}} \Lambda$ with $\mathbb{Z}[x, y, x^{-1}, y^{-1}]$, with $t \in \Lambda$ acting via xy). Note that $\delta(j_\lambda)(b_1) = \beta_{0n} = 1$ and $\beta_{pq} = 0$ unless $0 \le m < q \le n$.

Now $h_\Lambda T_M\rho_M(j_\lambda)(b_1) = e_\sharp(\eta_A \times T_M\rho_M(j_\lambda))(j_*(b_1)) = \Sigma\beta_{pq}t^p\psi_{p-q}$, where ψ_{-r} is the coefficient of t^{-r} in $Lexp(\lambda^{-1},\infty)$. Clearly $\psi_r = 0$ if $-n < r < 0$ and $\psi_{-n} = 1$, since $\lambda^{-1} = t^{-n}(1-\alpha t^{-1})^{-n}$. Hence $h_\Lambda T_M\rho_M(j_\lambda)(b_1) = \beta_{0n} = \delta(j_\lambda)(b_1)$, and so $h_\Lambda T\rho = \delta$, by linearity and functoriality.

Since δ and T are natural equivalences and h_Λ is injective, by Lemma 1, ρ and h_Λ are also natural equivalences. $\square$

Barge used a homomorphism similar to $T\rho$ in [Ba80] to embed $\mathbb{Q}(t)/\Lambda$ in $\mathbb{Q}[[t,t^{-1}]]$. The definition of ρ was influenced by the "trace" function used by Trotter to relate Seifert forms and Blanchfield pairings on a knot module M [Tr78].

§4. Other covering groups

The argument of Theorem 1 of [Hi98], using Shapiro's lemma and the finiteness criteria of Brown, readily extends to show that if X is a PD_n-complex then the homotopy fibre of a map $f : X \to S^1$ which induces an epimorphism on fundamental groups is finitely dominated (and hence is a PD_{n-1}-complex) if and only if it is homotopy equivalent to a complex with finite m-skeleton, where $m \geq \max\{2,[n/2]\}$. A simple induction extends this further to show that the cover associated to an epimorphism onto a virtually poly-Z group of Hirsch length r is a PD_{n-r}-complex, under the same finiteness hypothesis. However this hypothesis is *a priori* apparently stronger than that of Theorem 3. Can aspects of these arguments be combined to show that it suffices to assume $\chi(X) = 0$ and, say, $m \geq [(n-1)/2]$? In particular, suppose that M is a closed 4-manifold with $\chi(M) = 0$ and π is an extension of Z^r by a finitely generated infinite normal subgroup K, for some $r > 1$. Is M aspherical?

For more general quotient groups we must at present assume that X is itself aspherical, as in the next result.

Theorem 5. *Let π be a PD_n-group with an FP_r normal subgroup K such that $G = \pi/K$ is a PD_{n-r} group and $2r \geq n-1$. Then K is a PD_r-group.*

Proof. It shall suffice to show that $H^s(K;F) = 0$ for any free $\mathbb{Z}[K]$-module F and all $s > r$, for then $c.d.K = r$ and the result follows from Theorem 9.11 of [Bi]. Let $W = Hom_{\mathbb{Z}[K]}(\mathbb{Z}[\pi], F)$ be the $\mathbb{Z}[\pi]$-module coinduced from F. Then $H^s(K;F) \cong H^s(\pi;W) \cong H_{n-s}(\pi;\overline{W})$, by Shapiro's lemma and Poincaré duality. As a $\mathbb{Z}[K]$-module $\overline{W} \cong F^G$ (the direct product of $|G|$ copies of F), and so $H_q(K;\overline{W}) = 0$ for $0 < q \leq r$ (since K is FP_r), while $H_0(K;\overline{W}) \cong A^G$, where $A = H_0(K;F)$. Moreover $A^G \cong Hom_{\mathbb{Z}}(\mathbb{Z}[G], A)$ as a $\mathbb{Z}[G]$-module, and so is coinduced from a module over the trivial group. Therefere if $n - s \leq r$ the LHSSS gives $H^s(K;F) \cong H_{n-s}(G;A^G)$. Poincaré duality for G and another application of Shapiro's lemma now give $H^s(K;F) \cong H^{s-r}(G;A^G) \cong H^{s-r}(1;A) = 0$, if $s > r$. $\square$

Can a similar argument be applied in the non-aspherical case? The difficulty appears to be in computing $H_*(G;A^G)$ where A is a nontrivial G-module and A^G has the "diagonal" structure.

Corollary. *Let π be a PD_n-group with an FP_r normal subgroup K such that $G = \pi/K$ is a PD_{n-r} group and $2r \geq n-1$. Then K is a PD_r-group.* $\square$

This corollary may be used to reconcile the hypotheses of [Ka98] with those of [Ko99] and [Hi00] in their homotopy characterizations of complex surfaces which admit holomorphic submersions to complex curves. (Kapovich assumes the surface S is aspherical and that $\pi_1(S)$ is an extension of an orientable

PD_2-group by a finitely presentable normal subgroup, while Kotschick and Hillman assume that $\pi_1(S)$ is an extension of one orientable PD_2-group by another such and that $\chi(S)$ is the product of the Euler characteristics of these groups).

References

[Ba80] Barge, J. An algebraic proof of a theorem of J.Milnor, in *Proceedings of the Topology Symposium, Siegen, 1979*, (edited by U. Koschorke and W.D.Neumann), Lecture Notes in Mathematics 788, Springer-Verlag, Berlin - Heidelberg - New York (1980), 396-398.

[Ba80'] Barge, J. Dualité dans les revêtements galoisiens, Inventiones Math. 58 (1980), 101-106.

[Bi] Bieri, R. *Homological Dimension of Discrete Groups*, Queen Mary College Mathematics Notes, London (1976).

[Br75] Brown, K.S. A homological criterion for finiteness, Comment. Math. Helvetici 50 (1975), 129-135.

[Fa71] Farrell, F.T. The obstruction to fibering a manifold over a circle, Indiana Univ. J. Math. 21 (1971), 315-346.

[Hi81] Hillman, J.A. *Alexander Ideals of Links*, Lecture Notes in Mathematics 895, Springer-Verlag, Berlin - Heidelberg - New York (1981).

[Hi89] Hillman, J.A. A homotopy fibration theorem in dimension 4, Top. Appl. 33 (1989), 151-161.

[Hi] Hillman, J.A. *The Algebraic Characterization of Geometric 4-Manifolds*, London Mathematical Society Lecture Note Series 198, Cambridge University Press, Cambridge - New York - Melbourne (1994).

[Hi98] Hillman, J.A. On 4-dimensional mapping tori and product geometries, J. London Math. Soc. 58 (1998), 229-238.

[Hi00] Hillman, J.A. Complex surfaces which are fibre bundles, Top. Appl. 100 (2000), 187-191.

[HS] Hilton, P.J. and Stammbach, U. *A Course in Homological Algebra*, second edition, Graduate Texts in Mathematics 4, Springer-Verlag, Berlin - Heidelberg - New York (1997).

[Ka98] Kapovich, M. On normal subgroups in the fundamental groups of complex surfaces, preprint, the University of Utah (August, 1998).

[Ko99] Kotschick, D. On regular fibred complex surfaces, in *Proceedings of the Kirbyfest* (edited by J. Hass and M.Scharlemann), Geometry and Topology Monographs, Volume 2 (1999), 291-298.

[Ra95] Ranicki, A. Finite domination and Novikov rings, Topology 34 (1995), 619-632.

[St62] Stallings, J.R. On fibering certain 3-manifolds, in *Topology of 3-Manifolds and Related Topics* (edited by M.K.Fort, Jr), Prentice-Hall, Englewood Cliffs, N.J.(1962), 95-100.

[Tr78] Trotter, H.F. Knot modules and Seifert matrices, in *Knot Theory, Proceedings, Plans-sur-Bex, Switzerland 1977*, (edited by J.-C.Hausmann), Lecture Notes in Mathematics 685, Springer-Verlag, Berlin - Heidelberg - New York (1978),291-299.

[Wl] Wall, C.T.C. *Surgery on Compact Manifolds*, Academic Press, New York - London (1970).

[We87] Weinberger, S. On fibering four- and five-manifolds, Israel J. Math. 59 (1987), 1-7.