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THE EFFECT OF HIGHER-ORDER CURVATURE TERMS ON STRING QUANTUM COSMOLOGY

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Abstract

Several new results regarding the quantum cosmology of higher-order gravity theories derived from superstring effective actions are presented. After describing techniques for solving the Wheeler-DeWitt equation with appropriate boundary conditions, it is shown that this quantum cosmological model may be compared with semiclassical theories of inflationary cosmology. In particular, it should be possible to compute corrections to the standard inflationary model perturbatively about a stable exponentially expanding classical background.

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1. Classical Cosmological Space-times of Higher-Order Gravity Theories

Since solutions to the general relativistic field equations contain initial curvature singularities whenever the dominant energy condition is satisfied, one of the motivations for developing quantum cosmology has been the theoretical justification of the elimination of the singularities. Curvature singularities predicted by general relativity can be avoided by introducing boundary conditions in the path integral defining the quantum theory which restrict the integral over all four-geometries and matter fields to Riemannian metrics $g_{\mu\nu}$ on compact manifolds bounded by a three-dimensional hypersurface with metric h_{ij} and fields with specified values on the hypersurface [1]. They might also be eliminated in a quantum theory of gravity free from divergences at short distances, since the wavefunctions defined by the path integral of the theory may represent non-singular geometries at initial times.

Classical cosmological solutions to the equations of motion for several different types of theories containing higher-order curvature terms have been analyzed with regard also to the presence of singularities, and non-singular solutions have been obtained. Renormalizability has been obtained with the addition of quadratic terms in the action [2]. Moreover, any theory of superstrings consistent at the quantum level will have an effective action containing higher-order curvature terms. It has been shown, in particular, that dimensionally continued Lovelock invariants, giving rise to second-order field equations for the metric, arise in superstring effective actions.

According to the conventional usage of terms related to modifications of the Einstein-Hilbert action, a Lagrangian which has the form $L(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \partial_\alpha \partial_\beta g_{\mu\nu})$, and is a nonlinear function of the curvature tensor, represents a higher-order gravity theory [3], whereas an action containing third- and higher-order derivatives of the metric describes higher-derivative gravity theories.

To determine whether quantum cosmological models based on theories containing third- and higher-order derivatives of the metric or powers of the second derivative of the metric produce wave functions consistent with a non-singular geometry, without imposing a non-singular boundary condition at initial times, it is useful to start with an action which combines higher-order gravity with scalar fields and possesses singularity-free cosmological solutions [4][5]. At string tree-level and first-order in the α'-expansion of the compactified heterotic string effective action in four dimensions, the dynamics of the graviton, scalar field S and modulus field T can be described by the effective Lagrangian

$$\mathcal{L}_{eff.} \;=\; \frac{1}{2\kappa^2}R + \frac{DSD\bar{S}}{(S+\bar{S})^2} + 3\frac{DTD\bar{T}}{(T+\bar{T})^2} + \frac{1}{8}(Re\ S)R^2_{GB} + \frac{1}{8}(Im\ S)R\tilde{R} - \frac{1}{2}V(S,\bar{S},T,\bar{T}) \qquad (1)$$

If $Re\ T$, representing the square of the compactification radius, is set equal to a constant, and $Im\ T$ is set equal to zero, the kinetic term for the modulus field vanishes. In addition, defining the real part of the dilaton field to be $Re\ S \;=\; \frac{1}{g_4^2}\ e^{-\Phi}$ restricting attention to isotropic models for which $R\tilde{R} = 0$, and choosing units such that $\kappa \;=\; 1$, one obtains the action *

$$I \;=\; \int d^4x\ \sqrt{-g}\ \left[R \;+\; \frac{1}{2}(D\Phi)^2 \;+\; \frac{e^{-\Phi}}{4g_4^2}(R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2) \;-\; V(\Phi)\right] \qquad (2)$$

The Gauss-Bonnet invariant arises in this action, but it is multiplied now by the factor $\frac{e^{-\Phi}}{4g_4^2}$, where g_4 is the four-dimensional string coupling constant and Φ is the dilaton field, so that the integral is not a topological invariant. This quadratic curvature combination is precisely that required to remove ghost poles in the perturbative expansion of the propagator, and since the term is dynamical, the theory

* The sign convention in reference [6] is being used for the definition of $Re\ S$ and it is combined with the expression for $\mathcal{L}_{eff.}$ in reference [4]. The integral for the action I is deduced after multiplication by an overall factor of 2.

represents a modification of general relativity which is unitary and has improved renormalizability properties.

There are conformal transformations which map $f(R)$ actions to the Einstein-Hilbert action plus a scalar field with a potential term [7]-[13]. Other field redefinitions can be used to transform $f(R_{\mu\nu}R^{\mu\nu})$ to the Ricci scalar plus terms containing scalar fields and extra tensor modes [14]-[16]. The transformed theories should have the same renormalizability properties as the higher-order gravity theories, because field redefinitions do not alter the S-matrix. Such transformations are unnecessary in the present case since the field equations obtained from equation (2) contain at most two derivatives - in effect, although the action contains higher-order curvature terms, the addition of a boundary term is sufficient to eliminate second-order derivatives of the metric from the integral, so that only a conventional quantization of the theory is required, rather than the Ostrogradski method [3].

It is known that most classical string equations of motion without a dilaton field do not lead to inflation [17]. Thus, dilaton fields also have been included as they permit inflation, and a set of higher-order gravity theories with a dilaton field has been shown to produce the required inflationary growth of the Friedmann-Robertson-Walker scale factor [18].

An earlier analysis of R^2 theories [19] and C^2 theories [20] has shown that an R^2 term leads to particle production and inflation with minimal dependence on the initial conditions, while the C^2 term generates large anisotropy [21] and causes destabilization of positive Λ metrics. Inflation has been also derived from higher-derivative terms directly obtained as renormalization counterterms [22][23] without the inclusion of scalar or inflaton fields.

2. The Quantum Cosmology of Higher-Order Gravity Theories

Much of the initial work on higher-derivative and higher-order quantum cosmology has been developed with only curvature terms and no scalar field in the action. Nevertheless, scalar fields are probably necessary in a theory describing gravity and matter interactions, because they are required for renormalizability of gauge theories with massive spin-1 vector fields [24] and they result from the conformal rescaling of the metric in higher-order theories [7]-[13].

The quantum cosmology of standard gravity coupled to a scalar field has been investigated by many authors [25]-[28]. The non-zero vacuum expectation value of the scalar field in grand unified theories drives inflation in semi-classical cosmology, and again it is found to be useful in obtaining wave functions representing inflationary solutions in quantum cosmology.

The quantum cosmology of superstring and heterotic string effective actions in ten dimensions with curvature terms up to fourth order has been investigated previously [29]-[32]. With the dilaton and modulus fields included, the Wheeler-DeWitt equations for both theories, in the minisuperspace of metrics with different scale factors for the physical and internal spaces, have not been solved in closed form, because the coupling of the curvature and scalar fields leads to quartic terms in the derivatives of the scale factor in the Hamiltonian and fractional powers of the momenta [30]. For the superstring, the differential is only reducible to the form of a diffusion equation when the curvatures of the physical and internal spaces are set equal to zero, and the scalar field is set equal to a constant [32].

When there are quadratic curvature terms in the heterotic string effective action, the Hamiltonian cannot be expressed as a simple function of the canonical momenta, preventing a derivation of a directly solvable Wheeler-DeWitt equation. For the model in this paper, a pseudo-differential equation in closed form, obtained by using the Ostrogradski formalism for the heterotic string effective action (2), can be converted to a partial differential equation of higher order, which may be solved with appropriate boundary conditions [33].

The Hamiltonian is more directly obtained if one begins by adding a total derivative term to the Lagrangian to eliminate second-order derivatives and a different set of momenta conjugate to the scale factor and dilaton field only is used. While the first choice of momenta gives rise to a sixth-order Wheeler-DeWitt equation, the second set of momenta results in a Wheeler-DeWitt equation which reproduces the standard equation for gravity coupled to a scalar field in the limit that the quadratic term vanishes. The wave function can then be determined perturbatively in the dilaton coupling $\frac{e^{-\Phi}}{g_4^2}$.

Similarly, there is a preference for quantizing a theory that does not include higher-order curvature terms, because the absence of higher derivatives of the metric allows one to avoid superfluous degrees of freedom. However, if the required field redefinitions depend on the Riemann curvature tensor and its contractions, it is the set of kinetic terms for the extra scalar fields which now introduce the higher derivatives of the metric, implying that additional difficulties arising in quantization would not necessarily be circumvented for these types of higher-order theories.

While the coupling $\frac{e^{-\Phi}}{g_4^2}$ explicitly determines the strength of higher-order terms relative to the Ricci scalar, a factor of $\hbar$ can be restored at each order in the perturbative expansion of the S-matrix of the heterotic string sigma model. Since the $(n+1)-th$ loop computation for the sigma model produces the n^{th}-order contribution to the heterotic string effective action $I^{(n)}_{eff.}$, the limit $\hbar \to 0$ can be taken, after multiplication by a factor of $\frac{1}{\hbar}$, to obtain the classical action for gravity coupled to a scalar field. It is necessary to take the $\hbar \to 0$ limit because the $\frac{e^{-\Phi}}{g_4^2} \to 0$ limit eliminates the dilaton field kinetic term. Nevertheless, since additional factors of $\hbar$ and $\frac{e^{-\Phi}}{g_4^2}$ occur simultaneously at higher loops in the sigma model perturbation expansion, the Wheeler-DeWitt equation for the higher-order gravity theory can be regarded as a perturbation of the standard second-order Wheeler-DeWitt equation, with the solution being expanded in powers of $\frac{e^{-\Phi}}{g_4^2}$. This provides an approximate wave function for the heterotic string effective action which includes corrections to the wave function used to predict inflationary cosmology.

Given a fundamental theory at Planck scale with higher-order curvature terms, it is appropriate to consider a boundary located between the Planck era and the inflationary epoch where the predictions of quantum cosmology of the higher-derivative theory could be matched, in principle, to the predictions of the quantum theory of standard gravity coupled to matter fields. The inclusion of this boundary will have an effect on both the quantum cosmology of the more fundamental theory and the computation of radiative corrections to the semi-classical inflationary model.

The significance of the wave function depends on the stability of the classical background geometries which represent most probable configurations in the quantum cosmology of the model. Stability at the nonlinear level can be proven if the positive energy theorem holds, which requires that the space-time admits a Killing spinor that also must be a supersymmetry parameter. For the four-dimensional compactification of heterotic string theory, the Majorana condition on the Killing spinor leads to differential equation for the scale factor which is solved by a non-singular cosmological bounce solution. When this scale factor is substituted into the equation of motion for the dilaton, it will be shown that there is a solution for $\Phi(t)$ which increases at an approximately linear rate at large t. Thus, there is a stable background configuration derived from the heterotic string effective action, which describes the exponential expansion of the inflationary epoch and allows for the computation of perturbative corrections in the quantum cosmological model.

3. Quantum Cosmology for a Four-Dimensional Heterotic String Effective Action

The model (2) containing quadratic curvature terms and the dilaton field can be quantized with the Hamiltonian constraint being represented by the Wheeler-DeWitt equation. Since the solution to this equation generally requires a reduction in the number of degrees of freedom in the metric field, it is convenient to consider only a minisuperspace of Friedmann-Robertson-Walker metrics $g_{\mu\nu} = diag\left(1, -\frac{a^2(t)}{1-Kr^2}, -a^2(t)r^2, -a^2(t)r^2 sin\ \theta\right)$ with $K = 1$ (closed model), $K = 0$ (spatially flat model) or $K = -1$ (open model).

Homogeneity of the minisuperspace model implies that the fields are position-independent on foliations of the four-dimensional space-times and the action per unit volume is a one-dimensional integral

$$\frac{I}{\bar{\mathcal{V}}} = \int dt \left[(6a^2\ddot{a} + 6a\dot{a}^2 + 6aK) + \frac{1}{2}a^3\ \dot{\Phi}^2 + 6\frac{e^{-\Phi}}{g_4^2}\ddot{a}(\dot{a}^2 + K) - a^3V(\Phi)\right] \tag{3}$$

where $\bar{\mathcal{V}}$ is a time-independent volume factor given by $\frac{\mathcal{V}(t_f)}{a^3(t_f)}$, with $\mathcal{V}(t_f)$ being the three-dimensional volume of the spatial hypersurface at time t_f.

Since the field equations are not higher-derivative, the action can be modified by adding a boundary term to eliminate factors of $\ddot{a}$.

$$\begin{aligned}\frac{I'}{\bar{\mathcal{V}}} &= \frac{I}{\bar{\mathcal{V}}} - \int dt\ \frac{d}{dt}\ \left[2\frac{e^{-\Phi}}{g_4^2}\dot{a}(\dot{a}^2 + 3K) + 6a^2\dot{a}\ \right] \\ &= \int dt \left[6a(-\dot{a}^2 + K) + \frac{1}{2}a^3\dot{\Phi}^2 + 2\frac{e^{-\Phi}}{g_4^2}\dot{\Phi}\dot{a}(\dot{a}^2 + 3K) - a^3V(\Phi)\right]\end{aligned} \tag{4}$$

The conjugate momenta are then

$$P_a = -12a\dot{a} + 6\frac{e^{-\Phi}}{g_4^2}\dot{\Phi}(\dot{a}^2 + K) \qquad P_\Phi = a^3\dot{\Phi} + 2\frac{e^{-\Phi}}{g_4^2}\dot{a}(\dot{a}^2 + 3K) \tag{5}$$

Expanding in powers of $\frac{e^{-\Phi}}{g_4^2}$, expressions for $\dot{a}$ and $\dot{\Phi}$ to first order may be obtained

$$\begin{aligned}\dot{a} &\simeq -\frac{1}{12a}P_a + \frac{e^{-\Phi}}{g_4^2}\frac{1}{2a^4}P_\Phi\left(\frac{1}{144a}P_a\frac{1}{a}P_a + K\right) \\ \dot{\Phi} &\simeq \frac{1}{a^3}P_\Phi + \frac{e^{-\Phi}}{g_4^2}\frac{1}{6a^4}P_a\left(\frac{1}{144a}P_a\frac{1}{a}P_a + 3K\right)\end{aligned} \tag{6}$$

and the Hamiltonian is

$$\begin{aligned}H \simeq -6a&\left[\left[-\frac{1}{12a}P_a + \frac{e^{-\Phi}}{g_4^2}\frac{1}{2a^4}P_\Phi\left(\frac{1}{12a}P_a\frac{1}{12a}P_a + K\right)\right]^2 + K\right] \\ &+ 6\frac{e^{-\Phi}}{g_4^2}\ \left[\frac{1}{a^3}P_\Phi + \frac{e^{-\Phi}}{g_4^2}\frac{1}{6a^4}P_a\left(\frac{1}{12a}P_a\frac{1}{12a}P_a + 3K\right)\right] \\ &\quad \cdot \left\{\left[-\frac{1}{12a}P_a + \frac{e^{-\Phi}}{g_4^2}\frac{1}{2a^4}P_\Phi\left(\frac{1}{12a}P_a\frac{1}{12a}P_a + K\right)\right]^2 + K\right\} \\ &\quad \cdot \left[-\frac{1}{12a}P_a + \frac{e^{-\Phi}}{g_4^2}\frac{1}{2a^4}P_\Phi\left(\frac{1}{12a}P_a\frac{1}{12a}P_a + K\right)\right] \\ &+ \frac{a^3}{2}\ \left[\frac{1}{a^3}P_\Phi + \frac{e^{-\Phi}}{g_4^2}\frac{1}{6a^4}P_a\left(\frac{1}{12a}P_a\frac{1}{12a}P_a + 3K\right)\right]^2 + a^3V(\Phi)\end{aligned} \tag{7}$$

In a Lorentzian space-time, the differential operator representing the Hamiltonian is obtained by making the substitutions $P_a \to -i\frac{\partial}{\partial a}$ and $P_\Phi \to -i\frac{\partial}{\partial \Phi}$. Choosing the operator-ordering parameter to be equal to -1, so that $\left(\frac{1}{a}P_a\right)^2 \to -\frac{1}{a}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a}$, the $\hbar \to 0$ limit produces the equation

$$H_0\Psi \;=\; \left(\frac{1}{24}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a} \;-\; \frac{1}{2a^3}\frac{\partial^2}{\partial \Phi^2} \;-\; 6aK \;+\; a^3 V(\Phi)\right)\Psi \;=\; 0 \tag{8}$$

This is equivalent to the standard Wheeler-DeWitt equation for standard gravity plus a scalar field

$$\begin{gathered}\left[\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} \;-\; \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2} \;-\; a^2 U(a,\phi)\right]\Psi \;=\; 0 \\ U(a,\phi) \;=\; 1 \;-\; a^2\, V(\phi) \qquad\qquad V(\phi) \;=\; \textit{scalar potential}\end{gathered} \tag{9}$$

where, as above, the operator-ordering parameter p is set equal to -1, using the rescalings $a^2 \;\to\; \frac{1}{12\sqrt{K}}a^2$, $\Phi \;\to\; \frac{1}{2\sqrt{3}}\,\Phi$ and $V(\Phi) \;\to\; 72\; K^{\frac{3}{2}}V(\Phi)$.

While this choice of operator ordering is convenient for obtaining closed solutions to the Wheeler-DeWitt equation (8), other values of this parameter also can be used. The parameter p impacts on the regularity of the wave function $\Psi(\Phi, a)$ as $a \to 0$; when $p \geq 1$, regularity in the $a \to 0$ limit requires the no-boundary wavefunction [1], whereas the divergence in the tunneling wavefunction in the limit $a \to 0$ is regulated by a prefactor when $p \leq 0$ [34]. Non-singular solutions with Lorentzian signature will occur if regularity is maintained in this limit.

There are additional operator-ordering ambiguities at order $O\left(\frac{e^{-\Phi}}{g_4^2}\right)$, because terms containing the cube of $\frac{1}{a}P_a$ and the product of $\frac{e^{-\Phi}}{g_4^2}$ and P_Φ are included in the Hamiltonian. Suppose $\left(\frac{1}{a}P_a\right)^2$ is replaced by $-\frac{1}{a^{p+2}}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} \;=\; -\frac{1}{a}\frac{1}{a^{p+1}}\frac{\partial}{\partial a}\frac{1}{a^{1-(p+1)}}\frac{\partial}{\partial a}$ to obtain the second-order Wheeler-DeWitt equation (9), where $\frac{1}{a^{p+1}}\frac{\partial}{\partial a}\frac{1}{a^{1-(p+1)}}$ represents the ordering of the product of $\frac{1}{a}$ and $\frac{\partial}{\partial a}$. The second expression provides a way of obtaining higher powers of $\frac{1}{a}P_a$ through iteration, without the introduction of non-derivative terms. For example, $\left(\frac{1}{a}P_a\right)^3 \;\to\; (-i)^3\frac{1}{a}\left(\frac{1}{a^{p+1}}\frac{\partial}{\partial a}a^p\right)\left(\frac{1}{a^{p+1}}\frac{\partial}{\partial a}a^p\right)\frac{\partial}{\partial a} \;=\; i\frac{1}{a^{p+2}}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}$ represents a substitution for $\left(\frac{1}{a}P_a\right)^3$ which does not involve non-derivative terms and therefore has a form similar to the differential operator in equation (9).

More generally, powers of $\frac{1}{a}$ can be placed to the right of an operator product containing P_a, giving rise to non-derivative terms. Indeed, one can consider an arbitrary linear combination of the twenty operators $\mathcal{O}_i$, not all linearly independent, obtained by permutation of the factors in the product $\frac{1}{a}P_a\frac{1}{a}P_a\frac{1}{a}P_a$. This combination $\sum_{i=1}^{20}\;\alpha_i\mathcal{O}_i$ can be shown to be equal to $i\frac{1}{a^p}\frac{\partial}{\partial a}\frac{1}{a^q}\frac{\partial}{\partial a}\frac{1}{a^r}\frac{\partial}{\partial a}\frac{1}{a^s}$, where $p,\ q,\ r$ and s may be expressed in terms of the coefficients α_i, and $p+q+r+s=3$. Similarly, $\frac{e^{-\Phi}}{g_4^2}P_\Phi$ may be replaced by $-i\left[\frac{e^{-\Phi}}{g_4^2}\frac{\partial}{\partial\Phi} + t\frac{e^{-\Phi}}{g_4^2}\right]$, implying the existence of another operator-ordering parameter t in the Wheeler-DeWitt equation $H\Psi = 0$.

The powers $p,\ q,\ r$ and s are constrained by consistency with the algebra of supersymmetry constraints and hermiticity. While the potential in the heterotic string effective action is also determined by supersymmetry, it would be modified by the addition of any term not involving the derivative with respect to a in the expansion of $\left(\frac{1}{a}P_a\right)^3$.

Based on the substitutions

$$\begin{aligned}
\frac{1}{a}P_a &\rightarrow \frac{-i}{a^{p_1+1}}\frac{\partial}{\partial a}a^{p_1} \\
\left(\frac{1}{a}P_a\right)^2 &\rightarrow -\frac{1}{a^{p_2}}\frac{\partial}{\partial a}\frac{1}{a^{q_2}}\frac{\partial}{\partial a}\frac{1}{a^{r_2}} \qquad\qquad p_2+q_2+r_2 \;=\; 2 \\
\left(\frac{1}{a}P_a\right)^3 &\rightarrow \frac{i}{a^{p_3}}\frac{\partial}{\partial a}\frac{1}{a^{q_3}}\frac{\partial}{\partial a}\frac{1}{a^{r_3}}\frac{\partial}{\partial a}\frac{1}{a^{s_3}} \qquad p_3+q_3+r_3+s_3=3
\end{aligned} \tag{10}$$

the non-derivative term is

$$\begin{aligned}
&\left\{\frac{1}{24}\frac{r_2(q_2+r_2+1)}{a^3}+t\frac{e^{-\Phi}}{g_4^2}\left[\frac{1}{864a^9}\left[s_3(s_3+r_3+1)(q_3+r_3+s_3+2)+6(q_3+r_3+s_3)\right.\right.\right. \\
&\cdot(r_3+s_3(r_3+s_3+2))+\frac{3}{2}[(r_3+s_3(r_3+s_3+2))(q_3+r_3+s_3+1)+s_3(r_3+s_3+1)] \\
&\left.\left.\left.-6r_2(r_2+q_2+1)+30\right]+\frac{K}{4}\frac{(7-4p_1)}{a^5}\right]-\frac{e^{-\Phi}}{g_4^2}\frac{s_3(r_3+s_3+1)(q_3+r_3+s_3+2)}{1728\ a^9}\right\}\Psi
\end{aligned} \tag{11}$$

A specific operator-ordering, such as normal ordering, should be used uniformly for all of the terms in a Lagrangian field theory. In this case, setting p_1 equal to $\hat{p}$, it follows that $p_2=\hat{p}+1, q_2=1, r_2=-\hat{p}, p_3=\hat{p}+1, q_3=1, r_3=1, s_3=-\hat{p}$ so that the non-derivative term becomes

$$\begin{aligned}
&\left\{\frac{1}{24}\frac{\hat{p}(\hat{p}-2)}{a^3}+t\frac{e^{-\Phi}}{g_4^2}\left[\frac{1}{864\ a^9}\left(-\frac{17}{2}\hat{p}^3+\frac{81}{2}\hat{p}^2-56\hat{p}+\frac{93}{2}\right)+\frac{K}{4}\frac{(7-4\hat{p})}{a^5}\right]\right. \\
&\qquad\qquad\qquad\qquad \left.+\frac{e^{-\Phi}}{g_4^2}\frac{\hat{p}(\hat{p}-2)(\hat{p}-4)}{1728\ a^9}\right\}\Psi
\end{aligned} \tag{12}$$

When the factor of $\hbar$ is included in P_a, the first term in the expression (12) has the same order as $U(a,\Phi)$, whereas the second term represents an $O(\hbar)$ contribution. While this does represent a modification of $U(a,\Phi)$, invariance of the entire Lagrangian under supersymmetry transformations implies that consistency with the supersymmetry constraints would not be affected by rearrangement of terms in the Hamiltonian. The $O(1)$ part of the non-derivative term should be set equal to zero to minimize the correction to $U(a,\Phi)$, and this requires $\hat{p}=0,2$, when the operator-ordering (10) is applied uniformly. In terms of the original operator-ordering parameter p, these values correspond to $p=-1,2$. Non-derivative terms can be removed at $O(1)$ for more general values of p only if the substitution $\left(\frac{1}{a}P_a\right)^2\rightarrow -\frac{1}{a^{p+2}}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}$ is used instead of $\left(\frac{1}{a}P_a\right)^2\rightarrow -\frac{1}{a^{\hat{p}+1}}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a}a^{\hat{p}}$.

The heterotic string potential is

$$\begin{aligned}
V &= e^{-\mathcal{K}}\left[\mathcal{K}^\alpha\left(\mathcal{K}^{-1}\right)_\alpha^\beta\mathcal{K}_\beta\;+\;3\right] \\
\mathcal{K} &= ln(S\;+\;\bar{S})\;+\;3\;ln\;(T\;+\;\bar{T})\;-\;ln\;|\mathcal{W}(S)|^2 \qquad\qquad \mathcal{K}_\alpha \;=\; \frac{\partial\mathcal{K}}{\partial\phi^\alpha} \\
Re\ S &= \frac{e^{-\Phi}}{g_4^2} \qquad\qquad Re\ T \;=\; e^\sigma
\end{aligned} \tag{13}$$

where σ is set equal to a constant, σ_0, and $\{\phi^\alpha\}$ represents the fields S and T, with the other matter fields set equal to zero [35] - [38], and $\mathcal{W}(S)$ is the superpotential.

The equation $H\Psi = 0$ can be solved approximately by using $\epsilon \equiv \frac{e^{-\Phi}}{g_4^2}$ as an expansion parameter and noting that $\Psi = \Psi_0 + \epsilon\Psi_1$ is a solution to $(H_0 + \epsilon H_1)\Psi = 0$ to order $O(\epsilon)$ if $H_0\Psi_1 = -H_1\Psi_0$. Given the Hamiltonian (7), the first-order correction is

$$\begin{aligned}
&-\frac{1}{4}\frac{\partial}{\partial a}\frac{1}{a^4}\frac{\partial}{\partial \Phi}\left(-\frac{1}{144a}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a} + K\right) - \frac{1}{4a^3}\frac{\partial}{\partial \Phi}\left(-\frac{1}{144a}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a} + K\right)\frac{1}{a}\frac{\partial}{\partial a} \\
&+\frac{1}{2a^3}\frac{\partial}{\partial \Phi}\left(-\frac{1}{144a}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a} + K\right)\frac{1}{a}\frac{\partial}{\partial a} - \frac{1}{12}\left(\frac{\partial}{\partial \Phi} - 1\right)\frac{1}{a^4}\frac{\partial}{\partial a}\left(-\frac{1}{144a}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a} + 3K\right) \\
&\qquad - \frac{1}{12a}\frac{\partial}{\partial a}\left(-\frac{1}{144a}\frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a} + 3K\right)\frac{1}{a^3}\frac{\partial}{\partial \Phi}
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
H_1\Psi_0 \;=\; & \frac{1}{a^4}\left(\frac{K}{4} - \frac{1}{576a^4}\right)\frac{\partial\Psi_0}{\partial a} + \frac{1}{576a^7}\frac{\partial^2\Psi_0}{\partial a^2} - \frac{1}{1728a^6}\frac{\partial^3\Psi_0}{\partial a^3} \\
& + \frac{1}{a^5}\left(\frac{7K}{4} - \frac{35}{576a^4}\right)\frac{\partial\Psi_0}{\partial \Phi} + \frac{1}{24\,a^8}\frac{\partial^2\Psi_0}{\partial a\partial\Phi} - \frac{1}{64a^7}\frac{\partial^3\Psi_0}{\partial a^2\partial\Phi} + \frac{1}{864a^6}\frac{\partial^4\Psi_0}{\partial a^3\partial\Phi}
\end{aligned} \tag{15}$$

In the classically allowed range, the 'no boundary' wavefunction [1], for example, is

$$\Psi_{0_{NB}} \simeq exp\left(\frac{24K^{\frac{3}{2}}}{V}\right)\left(\frac{a^2V}{6K} - 1\right)^{-\frac{1}{4}} cos\left[\frac{24K^{\frac{3}{2}}}{V}\left(\frac{a^2V}{6K} - 1\right)^{\frac{3}{2}} - \frac{\pi}{4}\right] \tag{16}$$

and

$$\begin{aligned}
H_1\Psi_{0_{NB}} \;=\; & \frac{1}{a^3}\, exp\left(\frac{24K^{\frac{3}{2}}}{V}\right)\left[\frac{5}{K^{\frac{3}{2}}}\left(\frac{V}{144}\right)^2\left[\frac{a^2V}{6K} - 1\right]^{-\frac{7}{4}} - K^{\frac{3}{2}}\left(\frac{a^2V}{6K} + 2\right)\left[\frac{a^2V}{6K} - 1\right]^{\frac{1}{4}}\right] \\
& \qquad \cdot\, sin\left[\frac{24K^{\frac{3}{2}}}{V}\left(\frac{a^2V}{6K} - 1\right)^{\frac{3}{2}} - \frac{\pi}{4}\right] \\
& + \frac{1}{a^3}\, exp\left(\frac{24K^{\frac{3}{2}}}{V}\right)\left[\frac{5}{24}\left(\frac{V}{24K}\right)^3\left[\frac{a^2V}{6K} - 1\right]^{-\frac{13}{4}} + \frac{V}{48}\left(\frac{a^2V}{6K} - 2\right)\left[\frac{a^2V}{6K} - 1\right]^{-\frac{5}{4}}\right] \\
& \qquad \cdot\, cos\left[\frac{24K^{\frac{3}{2}}}{V}\left(\frac{a^2V}{6K} - 1\right)^{\frac{3}{2}} - \frac{\pi}{4}\right] + O\left(\frac{V'}{V}\right)
\end{aligned} \tag{17}$$

where terms involving the derivative of the potential with respect to Φ may be evaluated using the operator (15).

In the classically forbidden region [1][2],

$$\Psi_{0_{NB}} \simeq \frac{1}{2}\left[1 - \frac{a^2V}{6K}\right]^{-\frac{1}{4}} \cdot\, exp\left(\frac{24K^{\frac{3}{2}}}{V}\left(1 - \left(1 - \frac{a^2V}{6K}\right)^{\frac{3}{2}}\right)\right) \tag{18}$$

and

$$H_1\Psi_{0_{NB}} = \frac{1}{2a^3}\left\{-\frac{5}{24}\left(\frac{V}{24K}\right)^3\left[1 - \frac{a^2V}{6K}\right]^{-\frac{13}{4}} - \frac{5}{K^{\frac{3}{2}}}\left(\frac{V}{144}\right)^2\left[1 - \frac{a^2V}{6K}\right]^{-\frac{7}{4}}\right.$$
$$\left. + \frac{V}{48}\left(2-\frac{a^2V}{6K}\right)\left[1-\frac{a^2V}{6K}\right]^{-\frac{5}{4}} + K^{\frac{3}{2}}\left(\frac{a^2V}{6K}+2\right)\left[1-\frac{a^2V}{6K}\right]^{\frac{1}{4}}\right\} \tag{19}$$
$$\cdot\ exp\left(\frac{24K^{\frac{3}{2}}}{V}\left(1 - \left(1 - \frac{a^2V}{6K}\right)^{\frac{3}{2}}\right)\right) + O\left(\frac{V'}{V}\right)$$

Similarly, the tunneling wave function in the classically acceptable region [39]-[41] is

$$\Psi_{0_T} \simeq 2\ e^{\frac{i\pi}{4}}\left[\frac{a^2V}{6K} - 1\right]^{-\frac{1}{4}} exp\left[\frac{24K^{\frac{3}{2}}}{V}\left(1 - i\left(\frac{a^2V}{6K} - 1\right)^{\frac{3}{2}}\right)\right] \tag{20}$$

and

$$H_1\Psi_{0_T} = \frac{2}{a^3}\ e^{\frac{i\pi}{4}}\left\{\frac{5}{24}\left(\frac{V}{24K}\right)^3\left[\frac{a^2V}{6K}-1\right]^{-\frac{13}{4}} + \frac{5i}{K^{\frac{3}{2}}}\left(\frac{V}{144}\right)^2\left[\frac{a^2V}{6K}-1\right]^{-\frac{7}{4}}\right.$$
$$\left. + \frac{V}{48}\left(2-\frac{a^2V}{6K}\right)\left[\frac{a^2V}{6K}-1\right]^{-\frac{5}{4}} - iK^{\frac{3}{2}}\left(\frac{a^2V}{6K}+2\right)\left[\frac{a^2V}{6K}-1\right]^{\frac{1}{4}}\right\} \tag{21}$$
$$\cdot\ exp\left(\frac{24K^{\frac{3}{2}}}{V}\left(1 - i\left(\frac{a^2V}{6K}-1\right)^{\frac{3}{2}}\right)\right) + O\left(\frac{V'}{V}\right)$$

whereas the tunneling wave function in the classically forbidden region [41] is

$$\Psi_{0_T} \simeq \left[1 - \frac{a^2V}{6K}\right]^{-\frac{1}{4}}\left[exp\left(\frac{24K^{\frac{3}{2}}}{V}\left(1 - \left(1 - \frac{a^2V}{6K}\right)^{\frac{3}{2}}\right)\right)\right.$$
$$\left. + 2i\ exp\left(\frac{24K^{\frac{3}{2}}}{V}\left(1 + \left(1 - \frac{a^2V}{6K}\right)^{\frac{3}{2}}\right)\right)\right] \tag{22}$$

and

$$H_1\Psi_{0_T} = \frac{1}{a^3}\left\{-\frac{5}{24}\left(\frac{aV}{24K}\right)^3\left[1-\frac{a^2V}{6K}\right]^{-\frac{13}{4}} - \frac{5}{K^{\frac{3}{2}}}\left(\frac{V}{144}\right)^2\left[1-\frac{a^2V}{6K}\right]^{-\frac{7}{4}}\right.$$
$$\left. + \frac{V}{48}\left(2-\frac{a^2V}{6K}\right)\cdot\left[1-\frac{a^2V}{6K}\right]^{-\frac{5}{4}} + K^{\frac{3}{2}}\left(\frac{a^2V}{6K}+2\right)\left[1-\frac{a^2V}{6K}\right]^{\frac{1}{4}}\right\}$$
$$\cdot\ exp\left(\frac{24K^{\frac{3}{2}}}{V}\left(1-\left(1-\frac{a^2V}{6K}\right)^{\frac{3}{2}}\right)\right)$$
$$+ \frac{2i}{a^3}\left\{-\frac{5}{24}\left(\frac{aV}{24K}\right)^3\left[1-\frac{a^2V}{6K}\right]^{-\frac{13}{4}} + \frac{5}{K^{\frac{3}{2}}}\left(\frac{V}{144}\right)^2\left[1-\frac{a^2V}{6K}\right]^{-\frac{7}{4}}\right. \tag{23}$$
$$\left. + \frac{V}{48}\left(2-\frac{a^2V}{6K}\right)\left[1-\frac{a^2V}{6K}\right]^{-\frac{5}{4}} - K^{\frac{3}{2}}\left(\frac{a^2V}{6K}+2\right)\left[1-\frac{a^2V}{6K}\right]^{\frac{1}{4}}\right\}$$
$$\cdot\ exp\left(\frac{24K^{\frac{3}{2}}}{V}\left(1+\left(1-\frac{a^2V}{6K}\right)^{\frac{3}{2}}\right)\right) + O\left(\frac{V'}{V}\right)$$

Terms containing $V'(\phi)$ are negligible if $|V^{-1}V'(\phi)| << 1$ in Planck units, which holds for scalar potentials in standard inflationary models. In string cosmology, slow-roll inflation occurs because of the flatness of the potential in the chiral field directions [42], but it can be verified that the terms involving the derivative with respect to the dilaton, $V'(\Phi)$, are comparable in magnitude to the other terms in $H_1\Psi_0$ when $V(\Phi)$ does not contain an effective cosmological constant in addition to expressions multiplied by powers of $\frac{e^{-\Phi}}{g_4^2}$. While the $O(\frac{V'}{V})$ terms can be written down explicitly, they are omitted here so that compact expressions can be given for $H_1\Psi_{0_T}$ and $H_1\Psi_{0_{NB}}$. *

Nevertheless, in the parameter range where the derivative with respect to Φ can be ignored, the general solution to the equation $H_0\Psi_1 = -H_1\Psi_0$, given two linearly independent solutions of the homogeneous equation, $\left(\frac{1}{24}\frac{d}{da}\frac{1}{a}\frac{d}{da} - 6aK + a^3V(\Phi)\right)\Psi_0 \simeq 0$, would be

$$\Psi_1 \approx C_1\,\Psi_{0_1} + C_2\,\Psi_{0_2} - 24\,\Psi_{0_2}\int \Psi_{0_1}\frac{H_1\Psi_0}{W}\,a\,da + 24\,\Psi_{0_1}\int \Psi_{0_2}\frac{H_1\Psi_0}{W}\,a\,da \tag{24}$$

where $W = \Psi_{0_1}\frac{d}{da}\Psi_{0_2} - \Psi_{0_2}\frac{d}{da}\Psi_{0_1}$. Substituting $H_1\Psi_{0_{NB}}$ or $H_1\Psi_{0_T}$ into this formula and imposing the appropriate boundary conditions on Ψ_1 provides the correction of order $\frac{e^{-\Phi}}{g_4^2}\,\Psi_1$ to the standard wave function Ψ_0.

Amongst the solutions to the equations of motion of a one-loop effective Lagrangian, having the same form as $\mathcal{L}_{eff.}$ in equation (1) and containing only the dilaton field, are the homogeneous and isotropic solutions that begin with a Gauss-Bonnet phase $a(t) \sim e^{-\frac{\omega_1}{t}}$ and continue to the FRW phase $a(t) \sim t^{\frac{1}{3}}$ [43]. The stability of these solutions with respect to linearized tensor perturbations [44] is determined by the effective adiabatic index $\Gamma = \frac{2}{3}\left(1 - \frac{a\ddot{a}}{\dot{a}^2}\right)$ [45]. The Gauss-Bonnet phase is unstable since $\Gamma < \frac{5}{3}$, and the background geometry is eventually described by a stable FRW metric with $\Gamma = 2$.

Stability of the geometry, against perturbations of the metric that do not change the topology, follows from the positive-energy theorem, which is applicable when the metric admits Killing spinors. The requirement of covariance and spatial homogeneity implies the following form for the Killing spinor equation:

$$\nabla_\mu\,\eta + i\frac{k(t)}{2}\gamma_\mu\,\eta = 0 \tag{25}$$

for some function $k(t)$. Considering the spatial components of equation (25) and applying the commutator of the modified covariant derivative to the spinor η, the integrability relation [46] is $\frac{1}{4}R_{ij}{}^{\alpha\beta}\gamma_{\alpha\beta}\eta = \frac{k^2(t)}{2}\gamma_{ij}\eta$, implying that the four-dimensional space-time should be foliated by three-dimensional spaces satisfying the condition $R_{ijkl} = \frac{k^2(t)}{2}(g_{ik}g_{jl} - g_{il}g_{jk})$ and $k^2(t) = a^{-2}(K + \dot{a}^2)$.

The time-space component of the commutator condition is

$$\left(\frac{1}{2}R_{0i}{}^{0j}\gamma_{0j} + \frac{1}{4}R_{0i}{}^{jk}\gamma_{jk}\right)\eta = -\frac{i}{2}\dot{k}(t)\gamma_i\eta + \frac{k^2(t)}{2}\gamma_{0i}\eta \tag{26}$$

which implies that

$$\frac{1}{2}\frac{\ddot{a}(t)}{a(t)}\,\gamma_{0i}\eta = \frac{k^2(t)}{2}\gamma_{0i}\eta - \frac{i}{2}\dot{k}(t)\gamma_i\eta \tag{27}$$

Using the Dirac representation of the gamma matrices in four dimensions, $\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, and setting η equal to $c_1\begin{pmatrix}\eta_1\\0\end{pmatrix} + c_2\begin{pmatrix}0\\ \eta_2\end{pmatrix}$, it follows that

* The complete calculation of the terms containing $V'(\Phi)$ can be found in gr-qc/9911043.

$$c_1 \left(\frac{\ddot{a}(t)}{a(t)} \; - \; k^2(t) \; + \; i\dot{k}(t) \right) \begin{pmatrix} \eta_1 \\ 0 \end{pmatrix} \; + \; c_2 \left(-\frac{\ddot{a}(t)}{a(t)} \; + \; k^2(t) \; + \; i\dot{k}(t) \right) \begin{pmatrix} 0 \\ \eta_2 \end{pmatrix} \; = \; 0 \tag{28}$$

There are three types of solutions:

(i) $c_1, \; c_2 \neq \; 0, \; \dot{k}(t) \; = \; 0, \; \frac{\ddot{a}(t)}{a(t)} \; - \; k^2(t) \; = \; 0,$
(ii) $c_2 \; = \; 0, \; \frac{\ddot{a}(t)}{a(t)} \; - \; k^2(t) \; + \; i\dot{k}(t) \; = \; 0,$
(iii) $c_1 \; = \; 0, \; -\frac{\ddot{a}(t)}{a(t)} \; + \; k^2(t) \; + \; i\dot{k}(t) \; = \; 0.$

None of these conditions are satisfied by the scale factors $a(t) \sim e^{-\frac{\omega_1}{t}}$ or $a(t) \sim t^{\frac{1}{3}}$. This implies that the positive-energy theorem is not applicable to these background geometries, allowing for the possibility that they are unstable at a non-perturbative level. Moreover, these classical solutions may represent only local extrema of the action, providing sub-dominant contributions to the string path integral, since the absence of stability at the non-perturbative level suggests that there is tunneling from these solutions to more stable background geometries.

The Majorana condition implies that $c_1, \; c_2 \; \neq \; 0$ and the Killing spinor belongs to first category. When $K = 1$, both constraints are satisfied by the scale factor $a(t) = a_0 \; cosh(\frac{t}{a_0}), \; k(t) = \frac{1}{a_0}$, representing a cosmological bounce solution. When $K = 0$, the Killing spinor conditions require $a(t) = a_0 \; e^{\lambda t}$, and if $K = -1$, they imply that $a(t) = a_0 \; sinh(\frac{t}{a_0})$ or $a(t) = a_0 \; sin(\frac{t}{a_0})$.

Since the positive-energy theorem is applicable to these space-times, they will be stable at the non-perturbative level within the class of metrics representing the same topology. It may be noted that the $K = 0$ and $K = 1$ solutions do not have an initial singularity and that they provide adequate classical cosmological models through the inflationary epoch, as the scale factors $a(t)$ increase exponentially with time.

It is customary to attribute physical significance to a wave function only when there is an appropriate classical limit. While the metric with an initial Gauss-Bonnet phase and a perturbatively stable FRW phase represents a cosmological model which is realistic at later times, the stable, non-singular, exponentially expanding space-times are preferable as background geometries in this quantum cosmological model, because they describe the inflationary epoch, where quantum effects are still relevant.

The Euler-Lagrange equation of motion for Φ is

$$\ddot{\Phi} \; + \; 6\frac{e^{-\Phi}}{g_4^2} \left[\frac{\ddot{a}(K + \dot{a}^2)}{a^3} \; - \; \frac{\dot{a}^2(\dot{a}^2 + 3K)}{a^4} \right] \; + \; V'(\Phi) \; = \; 0 \tag{29}$$

With the scale factor $a(t) = a_0 \; cosh\left(\frac{t}{a_0}\right)$, and the heterotic string potential (13), this equation becomes

$$\begin{aligned} \ddot{\Phi} \; + \; \frac{6}{a_0^4}\frac{e^{-\Phi}}{g_4^2} \; sech^2\left(\frac{t}{a_0}\right)\left(1 - 2\; tanh^2\left(\frac{t}{a_0}\right)\right) \; - \; \frac{e^{-3\sigma_0}}{16}\frac{e^{-\Phi}}{g_4^2}\frac{d}{dS}\Bigg\{ \frac{1}{S} \; |\mathcal{W}(S)|^2 \\ \times \left[\frac{1}{S} - 2\frac{\mathcal{W}'(S)}{\mathcal{W}(S)}\right]^2 \left[-\frac{1}{S^2} + 2\frac{\mathcal{W}'(S)^2 - \mathcal{W}(S)\mathcal{W}''(S)}{\mathcal{W}(S)^2}\right]^{-1}\Bigg\} = 0 \end{aligned} \tag{30}$$

Using the superpotential $\mathcal{W}(S) = c + h\left(1 + \frac{3S}{b_0}\right) e^{-\frac{3S}{2b_0}}$ [47], the equation of motion becomes

$$\ddot{\Phi} + \frac{6}{a_0^4}\frac{e^{-\Phi}}{g_4^2} sech^2\left(\frac{t}{a_0}\right)\left(1 - 2\ tanh^2\left(\frac{t}{a_0}\right)\right) - \frac{e^{-3\sigma_0}}{16}\frac{e^{-\Phi}}{g_4^2}\left\{\left[-\frac{1}{S^2}\left[(c+h)+ \frac{3hS}{b_0}e^{-\frac{3S}{2b_0}}\right]^2 + \frac{1}{S}\left[-\frac{18hcS}{b_0^2}e^{-\frac{3S}{2b_0}} + \frac{3h^2}{b_0}\left(1+\frac{6S}{b_0}e^{-\frac{3S}{2b_0}}\right)\right]\right]\cdot\left[\frac{1}{S} - 2\frac{\mathcal{W}'(S)}{\mathcal{W}(S)}\right]^2 \right.$$
$$\cdot\left[-\frac{1}{S^2} + 2\frac{\mathcal{W}'(S)^2 - \mathcal{W}(S)\mathcal{W}''(S)}{\mathcal{W}(S)^2}\right]^{-1} + \frac{1}{S}\left[(c+h) + \frac{3hS}{b_0}e^{-\frac{3S}{2b_0}}\right]^2 \tag{31}$$
$$\cdot\left[2\left(\frac{1}{S} - 2\frac{\mathcal{W}'(S)}{\mathcal{W}(S)}\right) + 2\left(\frac{1}{S} - 2\frac{\mathcal{W}'(S)}{\mathcal{W}(S)}\right)^2\cdot\left(\frac{1}{S^2} + 2\frac{\mathcal{W}(S)\mathcal{W}''(S) - \mathcal{W}'(S)^2}{\mathcal{W}(S)^2}\right)^{-2}\right.$$
$$\left.\left.\left(-\frac{1}{S^3} + \frac{\mathcal{W}'''(S)}{\mathcal{W}(S)} - 3\frac{\mathcal{W}'(S)\mathcal{W}''(S)}{\mathcal{W}(S)^2} + 2\frac{\mathcal{W}'(S)^3}{\mathcal{W}(S)^3}\right)\right]\right\} = 0$$

which reduces to

$$\ddot{\Phi} + \frac{6}{a_0^4} sech^2\left(\frac{t}{a_0}\right)\left(1 - 2\ tanh^2\left(\frac{t}{a_0}\right)\right)\frac{e^{-\Phi}}{g_4^2} - \frac{27}{8}\frac{h^2 e^{-3\sigma_0}}{b_0^3}\frac{e^{-2\Phi}}{g_4^4} + O(e^{-3\Phi}) = 0 \tag{32}$$

when c is set equal to $-h$. Based on the leading-order term in $V'(\Phi)$, the dilaton field at large times is approximately

$$\begin{aligned}\Phi(t) &\simeq ln\left|\frac{3}{2a_0^2 g_4^2}\left(1+\sqrt{1+\frac{3}{8}\frac{a_0^2 h^2 e^{-3\sigma_0}}{b_0^3}}\right) cosh(\sqrt{C}t)\right| \\ C &= \frac{4}{a_0^2}\end{aligned} \tag{33}$$

and $\Phi(t) \rightarrow \sqrt{C}t + ln\left[\frac{3}{4a_0^2 g_4^2}\left(1+\sqrt{1+\frac{3}{8}\frac{a_0^2 h^2 e^{-3\sigma_0}}{b_0^3}}\right)\right] \equiv \sqrt{C}t + D$ as $t \rightarrow \infty$, representing the linear dilaton solution [48][49].

The equation of motion for $a(t)$ is

$$\begin{aligned}30\dot{a}^2 - 6K + 6\frac{e^{-\Phi}}{g_4^2}(\ddot{\Phi} - \dot{\Phi}^2)(\dot{a}^2 + K) - 12\left(a - \frac{e^{-\Phi}}{g_4^2}\dot{\Phi}\dot{a}\right)\ddot{a} \\ - 18\frac{e^{-\Phi}}{g_4^2}\frac{\dot{a}}{a}\dot{\Phi}(\dot{a}^2 + K) - \frac{3}{2}a^2\dot{\Phi}^2 + 3a^2 V(\Phi) = 0\end{aligned} \tag{34}$$

Given the string potential

$$V(\Phi) = \frac{g_4^2 e^{\Phi} e^{-3\sigma_0}}{16}\frac{\left[(c+h) + \frac{3h}{b_0}\frac{e^{-\Phi}}{g_4^2}e^{-3\frac{e^{-\Phi}}{2b_0 g_4^2}}\right]^2 \cdot \left[\left(c+h+\frac{9h}{b_0^2}\frac{e^{-2\Phi}}{g_4^4}\right)e^{-3\frac{e^{-\Phi}}{2b_0 g_4^2}}\right]^2}{\left[(c+h)^2 + \frac{3h(h+c)}{b_0}\frac{e^{-\Phi}}{g_4^2} - \frac{1}{4}\frac{h(50h+81c)}{b_0^2}\frac{e^{-2\Phi}}{g_4^4}\right]} \tag{35}$$

it is convenient to set $(c+h) = ke^{-\frac{\sqrt{C}}{2}t}$ instead of equating it to zero.

When $K = 1$ and $a(t) = a_0 \ cosh\left(\frac{t}{a_0}\right)$, to leading order, equation (34) implies $18 - \frac{3}{2}a_0^2 C + \frac{3}{16}a_0^2 g_4^2 e^{-3\sigma_0} k^2 e^D = 0$. While there is a positive solution for C when k is real, it does not equal $\frac{4}{a_0^2}$. A similar result occurs when $K = -1$. If $K = 0$ and $a(t) = a_0 \ e^{\lambda t}$, there is again a linear dilaton solution $\Phi(t) \sim ln \left|\frac{27}{4C}\frac{h^2 e^{-3\sigma_0}}{b_0^3 g_4^4} cosh(\sqrt{C}(t - t_0))\right|$. To leading order, the equation of motion for $a(t)$ gives rise to the relation $18\lambda^2 - \frac{3}{2}C + \frac{3}{16}g_4^2 e^{-3\sigma_0} k^2 e^D = 0$ which yields a positive value of C that can be adjusted with λ.

Since a linear growth for $\Phi(t)$ and an exponential scale factor $a(t)$, representing a stable classical background geometry, have been obtained from an approximate solution to the equations of motion for the action (4) derived from heterotic string theory, these properties should characterize configurations that are relatively more probable in the minisuperspace $\{a(t), \Phi(t)\}$. This allows one to compare directly predictions of the quantum cosmological model with standard inflationary cosmology. Moreover, supersymmetry ensures that the FRW universes with scale factors $a_0 \ cosh\left(\frac{t}{a_0}\right)$, $a_0 \ e^{\lambda t}$ and $a_0 \ sinh\left(\frac{t}{a_0}\right)$ are stable, so that inflation occurring in such exponentially expanding space-times would be terminated only when this symmetry is broken. Other scale factors, consistent with astrophysical observations, can be introduced consistently within this model for later epochs, once supersymmetry is broken.

The same techniques can be applied to other superstring effective actions with higher-order curvature terms, and again, the conjugate momenta and the Hamiltonian may be derived, with the solution to the Wheeler-DeWitt equation satisfying specified boundary conditions. Since the calculation of the wave function is equivalent to the evaluation of the path-integral over 4-metrics between initial and final times, the conclusions are consistent with the space-time foam picture, and theoretical expectation values of functions of the minisuperspace coordinates can be compared to observations of these cosmological variables.

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