

Advanced Methods of Mathematical Physics

MATH440x: Applied Mathematics Honours

Semester 2, 2019

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Due 11:59pm Saturday 14 September 2019

Submit your scanned or typeset answers to **TurnItIn** on Canvas. To ensure compliance with anonymous marking obligations, please do not include your name, only your SID should be present. As per University policy, no late work can be accepted.

1. Consider a sequence of monic polynomials $P_n(x)$, $0 \leq n \in \mathbb{N}$, with $P_0(x) = 1$, $P_1(x) = x$, which satisfy the orthogonality relation

$$\int_{-\infty}^{\infty} P_n(x)P_m(x) e^{-U(x)} dx = h_n \delta_{nm},$$

where $U(x) := x^6 - tx^4 - sx^2$, and t and s are parameters.

- (a) Using the above information, show that these polynomials satisfy a 3-term recurrence relation of the form

$$xP_n(x) = A_n P_{n+1}(x) + B_n P_n(x) + C_n P_{n-1}(x),$$

and find A_n , B_n , C_n explicitly in terms of h_n .

- (b) Derive structure relations, i.e., expressions of $\partial P_n / \partial x$, $\partial P_n / \partial t$, $\partial P_n / \partial s$ in terms of the above quantities and $\{P_k\}$, $\{h_k\}$, $k \in \mathbb{N}$.
- (c) Derive equations for $\partial h_n / \partial t$, $\partial h_n / \partial s$ in terms of the quantities in part (a).