# ADVANCED METHODS OF MATHEMATICAL PHYSICS

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## Summary

Much of our physical world is nonlinear. If you take two rulers and place one on top of another, the height of the combined object is the sum of the individual heights of each ruler. But whether you are looking at herds of bisons in a landscape, the viral load in an infective patient's bloodstream, or the interaction of black holes far away in the universe, it turns out the sum of individual components does not necessarily give a true measure of reality. An example is given in Figure (2) below.

To describe these systems, we need methods that apply to nonlinear mathematical models. In this course, I will cover theoretical methods (some exact, some in limits and others that are qualitative) to describe, solve and predict the results of such models.

Classical mathematical methods were developed for linear models. I will start with building blocks to describe models of semi-classical quantum mechanics and related orthogonal polynomials. These turn out to be generalizable to models that arise in modern physics, such as quantum gravity and random matrix theory. These lead naturally to integrable systems. We will cover examples of integrable systems and methods of solving these, before turning full circle to show how these also give rise to other special functions and polynomials.

(a) Sofia Kowalevsky (b) Paul Painlevé (c) Freeman Dyson (d) Martin Kruskal

Figure 1. Some mathematicians and physicists who shaped this area.

## Topics

1. Special Functions and ODEs
2. Orthogonal Polynomials
3. Equations on Lattices
4. Consistency and Integrability
5. Reduction of lattice equations
6. Riemann-Hilbert methods

## Assessment

$\boxed{\text{Assessments: } 60\% \text{ ; Exam: } 40\%}$

There will be two assignments, to be given out approximately in weeks 4 and 8 respectively. The exam is a take-home exam, to be done over a three-day period.

## Useful references

1. J. Hietarinta, N. Joshi and F.W. Nijhoff, *Discrete Systems and Integrability.* Cambridge University Press, Cambridge, 2016.
2. More to be added later.


Figure 2. Interacting waves near the Mexican coast have peak heights during collision that are not necessarily the sum of their individual heights. from [1, Figure 1 (b)].

## References

[1] M. J. Ablowitz and D. E. Baldwin, Phys. Rev. E**86** 036305 (2012).

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