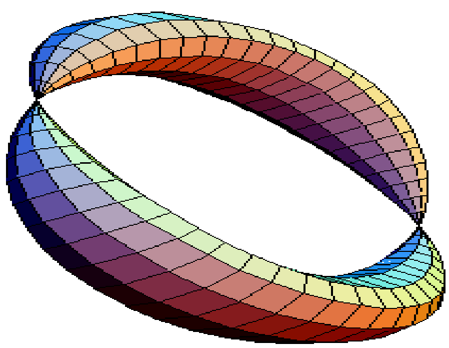
APPLIED MATHEMATICS HONOURS

Student Handbook 2021



School of Mathematics and Statistics

The University of Sydney

Contents

1 Entry Requirements

Entry requirements to the Honours Programme in Applied Mathematics vary slightly depending on whether the candidate is completing a Bachelor of Science (BSc) degree or a Bachelor of Advanced Studies (BAS) degree. They are usually based on satisfying the following criteria (and subject to approval by the Head of the School of Mathematics and Statistics):

1. **Faculty requirements**

   BSc: The candidate must have qualified for the Pass Degree in Mathematics (or discipline cognate with Mathematics) with a SCIWAM of at least 65, or

   BAS: Completed the Bachelor degree with two majors, at least one of which must be in Mathematics (or a discipline cognate with Mathematics) with a WAM of at least 65, and

2. **Mathematics requirements**

   BSc: The candidate must have completed 24 credit points of 3000 level or above mathematics units with an average mark of at least 65% for advanced level units and an average mark of at least 75% for mainstream units.

   BAS: The candidate must have completed a mathematics major with an average mark of at least 65% for advanced level mathematics units and an average mark of at least 75% for mainstream units.

3. **Essay/Project supervision**

   BSc/BAS: The candidate is required to find a prospective supervisor from among the Applied Mathematics staff, who is agreeable to supervise the candidate's essay or project in the candidate's chosen topic.

Students from institutions other than the University of Sydney must possess qualifications which are deemed equivalent to the above. There is some flexibility in these conditions; students not quite meeting them, but keen to pursue Honours in Applied Mathematics are invited to contact the Course Coordinator for advice. Students are expected to meet with prospective supervisors to discuss the potential for essays or projects and they are required to submit the Expression of Interest form before submitting the honours application.

Applications must be submitted to the Faculty of Science.

Application and enrolment information should also be obtained from the Faculty of Science, either in person of from their website. The Faculty will also provide AAM computations once the final third-year results are in.

Graduate Diploma in Science and MSc (Qualifying) applicants should see the Director of Postgraduate Studies before enrolling.

2 Course Administration

The Course Coordinator for Applied Mathematics Honours in 2021 is:

Prof Marek Rutkowski
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3 Structure of Applied Mathematics Honours

3.1 Coursework

Full-time students normally will enrol in 48 credit points, 24 of which will be coursework. Coursework requirements are the following:

1. **Core Units**
   Students are required to enrol in the following two 6 credit point core units
   - Semester 1: MATH 4411 Applied Computational Mathematics
   - Semester 2: MATH 4412 Advanced Methods in Applied Mathematics
2. **Selective Units**
   In addition to the core units, students should choose their two selective units from units offered by the School of Maths listed as 4000 level or higher, which have not already been taken for credit with the proviso that at most one unit labelled 5000 or higher be taken.

In addition to the units listed above, students may be allowed to take one unit from a related discipline. *Approval from the supervisor and the course coordinator needs to be obtained prior to enrolment.* The choice of coursework has to ensure that the student covers the proper material required for the student's project/essay.

3.2 Essay

In addition to the coursework, each student is also required to write an essay or project on an Applied Mathematics topic, under the supervision of a member of staff of the School of Mathematics and Statistics. This is considered to be the major aspect of the Honours Programme, and is detailed in Section .

3.3 Course Descriptions for the Core Units

MATH4411 Applied Computational Mathematics *(Semester 1)*
Prof G. Gottwald

The computational approach to study scientific problems serves a twofold purpose in mathematics. Numerical formulations and analysis help scientists in exploring complex systems, often giving rise to hitherto unknown phenomena which deserve further analytical treatment and understanding, or they can be used to verify existing mathematical theories.

On the other hand, the need for numerical simulations, for example in studying complex systems such as the ocean-atmosphere systems, requires new mathematics to allow for an efficient simulation. This course will highlight examples of this fruitful interplay of analytical mathematics and numerical computations.

The course will present a variety of numerical methods together with some of the underlying theory covering the numerical integration of partial differential equations, geometric integration, stochastic differential equations as well as some basic machine learning. When solving continuous time systems such as ordinary, partial or stochastic differential equations, we need to discretize time and space. How this affects the accuracy of our computations will be discussed, for example. The lectures will prepare the theory and background.

MATH2x63 and/or MATH3x76 or similar courses (such as COSC) provide a useful background for this course, but are not required.

MATH4412 Advanced Methods in Applied Mathematics *(Semester 2)*
Prof N. Joshi

Much of our physical world is nonlinear. If you take two rulers and place one on top of another, the height of the combined object is the sum of the individual heights of each ruler. But whether you are looking at herds of bisons in a landscape, the viral load in an infective patients bloodstream, or the interaction of black holes far away in the universe, it turns out the sum of individual components does not necessarily give a true measure of reality.

To describe these systems, we need methods that apply to nonlinear mathematical models. This course will cover theoretical methods (some exact, some in limits and others that are qualitative) to describe, solve and predict the results of such models.

Classical mathematical methods were developed for linear models. We will start with building blocks to describe models of semi-classical quantum mechanics and related orthogonal polynomials. These turn out to be generalizable to models that arise in modern physics, such as quantum gravity and random matrix theory. These lead naturally to integrable systems.

We will cover examples of integrable systems and methods of solving these, before turning full circle to show how these also give rise to other special functions and polynomials.

Topics:

- Introduction to nonlinear discrete models
- Special functions
- Orthogonal polynomials
- Random matrices
- Degree growth in discrete dynamical systems
- Riemann-Hilbert problems
- Higher dimensional problems
- Reductions
- Asymptotic estimates of solutions

Assessments: Everyone will be expected to participate in discussions of exercises in tutorials. There will be two assignments, to be given out approximately in weeks 4 and 8, respectively. The exam is a take-home exam, to be done over a three-day period. Assessments: participation in tutorials: 5%; assignments 55%; final examination: 40%.

3.4 Course Descriptions for Selective Units

MATH4413 Applied Mathematical Modelling *(Semester 1)*
TBA

Applied Mathematics harnesses the power of mathematics to give insight into phenomena in the wider world and to solve practical problems. Modelling is the key process that translates a scientific or other phenomenon into a mathematical framework through applying suitable assumptions, identifying important variables and deriving a well-defined mathematical problem. Mathematicians then use this model to explore the real-world phenomenon, including making predictions. Good mathematical modelling is something of an art and is best learnt by example and by writing, refining and analysing your own models. This unit will introduce you to some classic mathematical models and give you the opportunity to analyse, explore and extend these models to make predictions and gain insights into the underlying phenomena. You will also engage with modelling in depth in at least one area of application.

By doing this unit you will develop a broad knowledge of advanced mathematical modelling methods and techniques and know how to use these in practice. This will provide a strong foundation for applying mathematics and modelling to many diverse applications and for research or further study.

MATH4414 Advanced Dynamical Systems *(Semester 2)*
TBA

In applied mathematics, dynamical systems are systems whose state is changing with time. Examples include the motion of a pendulum, the change in the population of insects in a field or fluid flow in a river. These systems are typically represented mathematically by differential equations or difference equations. Dynamical systems theory reveals universal mechanisms behind disparate natural phenomena. This area of mathematics brings together sophisticated theory from many areas of pure and applied mathematics to create powerful methods that are used to understand and control the dynamical building blocks which make up physical, biological, chemical, engineered and even sociological systems.

By doing this unit you will develop a broad knowledge of methods and techniques in dynamical systems, and know how to use these to analyse systems in nature and in technology. This will provide a strong foundation for using mathematics in a broad sweep of applications and for research or further study.

DATA5441 Networks and High Dimensional Inference *(Semester 1)*
A/Prof E. Altmann

In our interconnected world, networks are an increasingly important representation of datasets and systems. This unit will investigate how this network approach to problems can be pursued through the combination of mathematical models and datasets. You will learn different mathematical models of networks and understand how these models explain non-intuitive phenomena, such as the small world phenomenon (short paths between nodes despite clustering), the friendship paradox (our friends typically have more friends than we have), and the sudden appearance of epidemic-like processes spreading through networks. You will learn computational techniques needed to infer information about the mathematical models from data and, finally, you will learn how to combine mathematical models, computational techniques, and real-world data to draw conclusions about problems. More generally, network data is a paradigm for high-dimensional interdependent data, the typical problem in data science.

By doing this unit you will develop computational and mathematical skills of wide applicability in studies of networks, data science, complex systems, and statistical physics.

Units previously labelled as 39XX and now labelled as 40XX are also available to be taken as selective units. Descriptions of 40xx units can be found on the School's website

http://www.maths.usyd.edu.au/u/UG/SM

3.5 Important Dates in 2021

Semester 1: 1 March–4 June 2021

- **Seminar: Week of 26-30 April 2021 (week 8)**
- **Essay Submission: 10 pm on Monday, 31 May 2021 (week 13)**
  An electronic file (pdf format) and three typed and bound copies of the Essay/Project are to be handed in to the Applied Mathematics Honours Course Coordinator by this date and time. Note the electronic pdf file must be submitted to the Honours Coordinator and uploaded on Canvas by this deadline.

Semester 2 (TBA)

- **Seminar: Week 8**
- **Essay Submission: 10 pm on Monday in week 13**
  An electronic file (pdf format) and three typed and bound copies of the Essay/Project are to be handed in to the Applied Mathematics Honours Course Coordinator by this date and time. Note the electronic pdf file must be submitted to the Honours Coordinator and uploaded on Canvas by this deadline.

3.6 Assessment Procedures

The Honours mark for each student is computed based on the following:

- 50% for the Project/Essay assessment;
- 50% for 4 courses (12.5 % for each).

The marking scale for Honours is significantly different from the undergraduate marking scale at the University of Sydney. The Essay/Project, in addition to all the fourth-year courses, will be marked with this scale in mind.

Grade of Honours	Faculty-Scale
First Class, with Medal	95–100
First Class (possibly with Medal)	90–94
First Class	80–89
Second Class, First Division	75–79
Second Class, Second Division	70–74
Third Class	65–69
Fail	00–64

4 The Essay/Project

A significant part of the honours year is the completion of an honours essay or project by each student. Each student must choose an essay/project supervisor who is willing to supervise the student's chosen topic for the essay or project. Essay/project topics and supervisors should be finalised by the beginning of the first semester, so that students can commence work immediately on their essays/projects. The following list shows the main applied mathematics research areas:

- Dynamical Systems
- Geophysical and Astrophysical Fluid Dynamics
- Industrial and Biomedical Modelling
- Integrable Systems
- Mathematical Biology

For detailed information about these areas and the corresponding staff, see the website https://www.sydney.edu.au/science/our-research/research-areas/mathematics-and-statistics /applied-mathematics-research-group.html

4.1 Assessment of the Essay/Project

Essay/Project will be marked by three examiners, one of whom is the supervisor, and each marking will therefore constitute 30% of the final essay/project mark. Note that the assessment also includes a one page report submitted at the end of the first semester. Although marking schemes may differ, marks will generally awarded for:

(i) selection and synthesis of source material;
(ii) evidence of understanding;
(iii) evidence of critical ability;
(iv) clarity, style and presentation;
(v) mathematical and/or modelling expertise.

The essay or project will be marked according to the following.

- 90% for the final written report
- 10% for a seminar presentation on the essay/project

The seminar is an opportunity for each student to present the material of his or her Essay or Project to a mathematically literate audience. The seminar talk will usually be of 25 minutes duration, with an additional 5 minutes set aside for questions. The presenter of the best talk will be awarded the Chris Cannon Prize.

4.2 Essay/Project Guidelines

- The student should consult the supervisor on a regular basis, preferably at least once a week. This is the student's responsibility.
- A realistic schedule for work on the essay or project should be drawn up at an early stage, and adhered to as closely as possible. If it proves necessary to modify the original plans, a revised schedule should be drawn up after discussion with the supervisor.
- At the end of Semester 1, a one page report is to be submitted to the Honours coordinator. This report includes a half page description about the students aim/scope of the project/essay and a half page description about what the student has achieved in semester 1 and what the student wants to achieve in semester 2. This report has to be approved by the supervisor before submission.
- The essay/project should be both a discursive and a critical account of the selected topic. It should be written at a level that an expert Applied Mathematician can be expected to understand, though he/she need not be an expert in the field covered. The work must contain substantial mathematical content.
- The essay/project should be based on some four to six original primary source articles, which themselves represent a substantial contribution to the topic. Secondary sources, such as books, review papers, etc., should also be consulted and cited.
- Original research is not essential.
- The length of the essay/project should be between 40 to 60 typed A4 pages. Only in exceptional circumstances, and after consultation with the supervisor, should the essay exceed 60 pages. This number includes all figures, contents pages, tables, appendices, etc. Computer programs essential to the work should be included (with adequate commentary) as additional material in appendices.
- Students should be careful to provide full and correct referencing to all material used in the preparation of essays and projects. Be explicit in stating what is your contribution and what is someone else's contribution. Avoid quoting verbatim unless reinforcing an important point.
- Students are advised to read the pamphlet entitled "*Guide to Essay Writing for Science Students*" available from the Science Faculty Office.
- Students are recommended to use LaTeX in typesetting their Essays/Projects. Additional information on LaTeX is available from the course coordinator.
- Students who have worked on their essays or project topics as Vacation Scholars are required to make a declaration to that effect in the preface of their essay/project.

4.3 Suggested Topics for the Essay/Project

The following is a list of possible essay/project topics for Applied Mathematics Honours students in 2020. Prospective students interested in any of these topics are encouraged to discuss them with the named supervisors as early as possible. However, this list is not exhaustive. Students may wish to suggest their own topics for essays or projects. Before commencing work, however, each student must find a member of staff who will agree to supervise the essay/project. For topics other than those listed below, the student and supervisor must submit a brief written outline of the proposed project or essay for approval by the Course Coordinator.

Monte Carlo methods in triangulation problems (Project)
Supervisor: A/Prof E. Altmann (Carslaw 615; phone 9351-2448)

The goal of this project is to investigate how Markov Chain Monte Carlo methods can be used to optimise triangulations of manifolds with a range of different applications in mind. After reviewing the known results for simple configurations (in low dimensions), we will focus on computational methods to efficiently find triangulations with good properties. Within a Monte Carlo framework, we will investigate the efficiency of different proposal steps such as moves that merge and subdivide triangles or flip edges (so-called bistellar moves). This project involves programming, it lies in the intersection between Applied and Pure mathematics, and will be co-supervised by Dr J. Spreer. Related works: Aste, Gramatica, and Di Matteo, "Random and frozen states in complex triangulations," *Philosophical Magazine* 92:1-3 (2001), 246–254 and Björner and Lutz, "Simplicial manifolds, bistellar flips and a 16-vertex triangulation of the Poincaré homology 3-sphere," *Experimental Mathematics* 9(2) (2000), 275–289.

Statistical laws, information theory, and natural language (Project)
Supervisor: A/Prof E. Altmann (Carslaw 615; phone 9351-2448)

How dissimilar are texts from different authors, epochs, or literary genres? Information theory provides a quantitative and mathematically well-defined answer to these questions. For instance, suppose a single (random) word is shown to you and your task is to guess from which of two texts this word was extracted from. The dissimilarity between the two texts is then quantified as the (expected) amount of (Shannon) information you obtain from the word to solve your task. While these and other natural-language questions have long been a main motivation for information theory, the recent availability of large amounts of texts (e.g., on the Internet and due to the digitization of information) opens new possibilities of investigations and applications. In this project we will explore the consequences of statistical regularities observed in large datasets of written language (e.g., Zipf's law) to the computation of similarity measures. Example of questions we will address from an information-theoretic perspective are: how similar are spoken and written English? Is Australian English more similar to British or to American English? Given a series of observations of two sources, how can we quantify the extent into which one source is affected by the other? This project requires programming skills and the interest in applications of statistical methods.

A complex-network approach to rare diseases (Project)
Supervisor: A/Prof E. Altmann (Carslaw 615; phone 9351-2448)

The relationship between different diseases can be represented as networks that capture both common biological pathways and phenotypes of the diseases [1,2]. This project will investigate rare diseases using complex-network models and methods applied to read data. We will characterize the main properties of the different networks and consider in which extent a multi-layer representation of the networks is essential to explain its characteristics. We will then use different approaches to identify network communities, e.g., by using Stochastic Block Models [3]. This will provide a probabilistic description of the problem that allow us not only to cluster genes and diseases but also to attribute the probability of (missing) links. In turn, this provides a direct connection to domain-specific problems such as the identification of common genes or missing/spurious phenotypes. Requirements: basic knowledge of networks and experience with coding (preferentially Python). This project will include collaborations with Prof A. Zankl (Medical Genetics) and Dr L. Azizi (Statistics).

[1] The Human Disease network, K.-Il Goh et al., PNAS 2007.

[2] Uncovering disease-disease relationships through the incomplete interactome, J. Menche et al., Science 2015.

[3] Bayesian stochastic blockmodeling, T. P. Peixoto, Advances in network clustering and blockmodeling 2019.

Fractals in transiently chaotic systems (Project)
Supervisor: A/Prof E. Altmann (Carslaw 615; phone 9351-2448)

The most interesting dynamics of dynamical systems often happens during the transient time it needs to reach an asymptotic state which is often trivial (e.g., the rest state of autonomous systems with friction). This project will investigate how chaos and fractality are defined and quantified during such transients. After a revision of the main concepts of transient chaos theory, it will focus on one of two areas of current research: (i) the appearance of fractal basin boundaries in undriven systems showing “double transient chaos”; or (ii) optical billiards in which the intensity of light is partially reflected and partially transmitted. The challenge in both cases is to generalize and apply measures of fractal dimension used in traditionally transiently-chaotic systems (in particular, the uncertainty dimension). The project involves a combination of analytical calculations and numerical simulations.

Data and models in complex networks (Project)
Supervisor: A/Prof E. Altmann (Carslaw 615; phone 9351-2448)

The field of complex networks combines mathematical models for the generation of networks with the empirical analysis of network data. Crucial in this approach is the identification of statistical regularities that appear “universally” in various types of networks and that serve as motivation to consider generative models that explain these regularities. Paradigmatic examples are the small world properties and the scale-free distribution of node degrees.

The modern availability of large amounts of data that can be represented in form of networks bring new opportunities and challenges for the mathematical modeling of complex networks. In particular, a more rigorous connection between the statistical analysis of the data and the mathematical generative models is crucial. The goal of this project is to re-frame previously established connections between statistical analysis and generative models as a (Bayesian) inference problem on suitably defined probabilistic network models. This project will be co-supervised with Dr L. Azizi (Statistics).

Analysis and simulation of simpler mechanical systems (Project)
Supervisor: Prof H. Dullin (Carslaw 714; phone 9351-4083)

This project is about the dynamics of mechanical system with few degrees of freedom (for example the double or triple pendulum, the Foucault pendulum, a spinning top, the Levitron, ... I am open to your suggestions, please come talk to me if you have further ideas/suggestions). The goal of the project is to get a good understanding of the dynamics of the chosen systems. Depending on the example different techniques will be applied, ranging from analytical computations to the numerical integration of the problem.

Simulating a closed chain of planar rigid bodies (Project)
Supervisor: Prof H. Dullin (Carslaw 714; phone 9351-4083)

A chain of planar rigid bodies is a simple mechanical system with n segments connected by joints that allow free rotation. Connecting the first segment to the last by another joint gives a closed chain. Since the distance between the joints is fixed the closed chain has n degrees of freedom. Reduction by translations and rotations leaves $n - 3$ degrees of freedom specifying the shape. For certain parameters the dynamics of this system is chaotic in the sense of Anosov. The goal of the project is the numerical solution of the equations of motion for the case of the five-gon, with the ultimate goal of finding many periodic orbits.

Rotation number near the 1:3 resonance (Project)
Supervisor: Prof H. Dullin (Carslaw 714; phone 9351-4083)

Consider a Hamiltonian system with two degrees of freedom near an equilibrium point with frequencies (of the harmonic oscillator approximation) in 1:3 resonance. Every such system can be approximated by the so called resonant Birkhoff normal form. This resonant Birkhoff normal form is an integrable system, and the goal of the project is to compute the rotation number (i.e. the ratio of frequencies) of the motion for general initial condition near the equilibrium point. These are given by complete elliptic integrals, which need to be analysed.

Symplectic integration of the regularised planar circular restricted 3 body problem (Project)
Supervisor: Prof H. Dullin (Carslaw 714; phone 9351-4083)

The restricted three body problem describes the motion of a test particle in the field of two heavy masses rotating around each other in circular orbits. The problem has a singularity when the test particle collides with either of the other masses. The collision can be regularised, such that the solutions are defined for all times. The goal of the project is to construct and implement a symplectic integration method for the regularised problem. This can then be used to study periodic orbits in this chaotic system, in particular including collision orbits.

Data-driven modelling: Finding models for observations in finance and climate (Project)
Supervisor: Prof G. Gottwald (Carslaw 625; phone 9351-5784)

When given data, which may come from observations of some natural process or data collected form the stock market, it is a formidable challenge to find a model describing those data. If the data were generated by some complex dynamical system one may try and model them as some diffusion process. The challenge is that even if we know that the data can be diffusive, it is by no means clear on what manifold the diffusion takes place. This project aims at applying novel state-of-the-art methods such as diffusion maps and nonlinear Laplacian spectral analysis to determine probabilistic models. You will be using data from ice cores encoding the global climate of the past 800kyrs as well as financial data. In the latter case you might be able o recover the famous Black-Scholes formula (but probably not). This project requires new creative ideas and good programming skills.

Optimal power grid networks and synchronisation (Project)
Supervisor: Prof G. Gottwald (Carslaw 625; phone 9351-5784)

Complex networks of coupled oscillators are used to model systems from pacemaker cells to power grids. Given their sheer size we need methods to reduce the complexity while retaining the essential dynamical information. Recent new mathematical methodology was developed to describe the collective behaviour of large networks of oscillators with only a few parameters which we call "collective coordinates." This allows for the quantitative description of finite-size networks as well as chaotic dynamics, which are both out of reach for the usually employed model reduction methods.

You will apply this methodology to understand causes of and ways to prevent glitches and failure in the emerging modern decentralised power grids. As modern societies increase the share of renewable energies in power generation the resulting power grid becomes increasingly decentralised. Rather than providing a power supply constant in time, the modern decentralised grid generates fluctuating and intermittent supply. It is of paramount importance for a reliable supply of electric power to understand the dynamic stability of these power grids and how instabilities might emerge. A reliable power-grid consists of well-synchronised power generators. Failing to assure the synchronised state results in large power outages as, for example, in North America in 2003, Europe in 2006, Brazil in 2009 and India in 2012 where initially localised outages cascade through the grid on a nation-wide scale. Such cascading effects are tightly linked to the network topology. Modern power-grids face an intrinsic challenge: on the one hand decentralisation was shown to favour synchronisation in power grids, on the other hand decentralised grids are more susceptible to dynamic perturbations such as intermittent power supply or overload.

The project uses analytical methods as well as computational simulation of models for power grids. You will start with a simple network topology and then, if progress is made, use actual power grid topologies.

Data assimilation in numerical weather forecasting and climate science (Project)
Supervisor: Prof G. Gottwald (Carslaw 625; phone 9351-5784)

Data assimilation is the procedure in numerical weather forecasting whereby the information of noisy observations and of an imperfect model forecast with chaotic dynamics, we cannot trust, is combined to find the optimal estimate of the current state of the atmosphere (and ocean). Data assimilation is arguably the most computationally costly step in producing modern weather forecasts and has been topic of intense research in the last decade. There exists several approaches, each of which with their own advantage and disadvantages. Recently a method was introduced to adaptively pick the best method to perform data assimilation. This method employs a switch which, although it seems to work, has not been linked to any theoretical nor physical properties of the actual flow. This project will be using toy models for the atmosphere to understand the witch with the aim of improving the choice of the switching parameter.

Networks of coupled oscillators (Project)
Supervisor: Prof G. Gottwald (Carslaw 625; phone 9351-5784)

Many biological systems are structured as a network. Examples range from microscopic systems such as genes and cells, to macroscopic systems such as fireflies or even an applauding audience at a concert. Of paramount importance is the topography of such a network, i.e., how the nodes, let's say the fireflies, are connected and how they couple. Can they only see their nearest neighbours, or all of them. Are some fireflies brighter than others, and how would that affect the overall behaviour of a whole swarm of fireflies? For example, the famous 'only 6 degrees of separation'-law for the connectivity of human relationships is important in this context.

In this project we aim to understand the influence of the topography of such a network. Question such as: How should a network be constructed to allow for maximal synchronization will be addressed. This project requires new creative ideas and good programming skills.

Weather derivatives (Project)
Supervisor: Prof G. Gottwald (Carslaw 625; phone 9351-5784)

Weather derivatives were introduced to safe guard economies which severely depend on the weather. The question we propose in this project is whether the value of weather derivatives has any information about the actual weather which will occur; in other words: Can they be used to do weather forecasting. The trader probably uses different forecast models from different weather centres across the world to determine the value; can the value of a weather derivative be understood as a multi-model forecast. We will use real data. This project requires new creative ideas and good programming skills. Some exposure to statistical ideas and Bayesian methodology would be desirable but is not essential.

Discrete soliton equations (Project)
Supervisor: Prof N. Joshi (Carslaw 629; phone 9351-2172)

Famous PDEs such as the Korteweg-de Vries equation (which have soliton solutions) have discrete versions (which also have soliton solutions). These discrete versions are equations fitted together in a self-consistent way on a square, a 3-cube or an N-dimensional cube. These have simple, beautiful geometric structures that provide information about many properties: solutions, reductions to discrete versions of famous ODEs, and deeper aspects such as Lagrangians. This project would consider generalisations of such structures and/or properties of the solutions, such as finding their zeroes or poles.

Integrable discrete or difference equations (Essay or Project)
Supervisor: Prof N. Joshi (Carslaw 629; phone 9351-2172)

The field of integrable difference equations is only about 20 years old, but has already caused great interest amongst physicists (in the theory of random matrices, string theory, or quantum gravity) and mathematicians (in the theory of orthogonal polynomials and soliton theory). For each integrable differential equation there are, in principle, an infinite number of discrete versions. An essay in this area would provide a critical survey of the many known difference versions of the classical Painlevé equations, comparisons between them, and analyse differing evidence for their integrability. Project topics would include the derivation of new evidence for integrability. The field is so new that many achievable calculations remain to be done: including derivations of exact solutions and transformations for the discrete Painlevé equations.

Exponential asymptotics (Project)
Supervisor: Prof N. Joshi (Carslaw 629; phone 9351-2172)

Near an irregular singular point of a differential equation, the solutions usually have divergent series expansions. Although these can be 'summed' in some way to make sense as approximations to the solutions, they do not provide a unique way of identifying a solution. There is a hidden free parameter which has an effect like the butterfly in chaos theory. This problem has been well studied for many classes of nonlinear ODEs but almost nothing is known for PDEs and not much more is known for difference equations. This project would include studies of a model PDE, like the famous Korteweg-de Vries equation near infinity, or a difference equation like the string equation that arises in 2D quantum gravity.

Cellular automata (Essay or Project)
Supervisor: Prof N. Joshi (Carslaw 629; phone 9351-2172)

Cellular automata are mathematical models based on very simple rules, which have an ability to reproduce very complicated phenomena. (If you have played the Game of Life on a computer, then you have already seen automata with complicated behaviours.) This project is concerned with the mathematical analysis of their solutions, which lags far behind corresponding developments for differential or difference equations.

In this project, we will consider a family of cellular automata called parity filter rules, for which initial data are given on an infinite set. For example, consider an infinitely long train of boxes, a finite number of which have a ball inside, whilst the remainder are empty. At each time step, there is a simple rule for moving the leftmost ball in a box to the next empty box on the right. Continue until you have finished updating all nonempty boxes in the initial train. (Try this out for yourself with adjacent boxes with three balls, followed by two empty boxes and then two boxes with balls inside. What do you see after one update? Two updates?) It turns out that these box-and-ball systems replicate solitons, observed in solutions of integrable nonlinear PDEs. In this project, we will consider how to derive parity filter rules from nonlinear difference equations, and how to analyse their solutions. One direction for the project is to analyse the solutions as functions of initial data. Another direction is to develop ways to describe long-term behaviours.

Modelling the evolution of human post-menopausal longevity and pari bonding

(Project)
Supervisor: A/Prof P. Kim (Carslaw 621; phone 9351-2970)

A striking contrast between humans and primates is that human lifespans extend well beyond the end of the female reproductive years. Natural selection favours individuals with the greatest number of offspring, so the presence of a long female post-fertile period presents a challenge for understanding human evolution.

One prevailing theory that attempts to explain this paradox proposes that increased longevity resulted from the advent of grandmother care of grandchildren. We have developed preliminary age-structured PDE models and agent-based models to consider the intergenerational care of young proposed by this Grandmother Hypothesis. The project will involve extending the models to consider whether the presence of grandmothering could increase the optimum human longevity while simultaneously maintaining a relatively early end of fertility as seen in humans (and killer whales).

Analytical approaches will involve developing numerical schemes for the PDEs and analytically and numerically studying the steady state age distributions and growth rates of the populations with and without grandmothering and under different life history parameters, e.g. longevity and end of fertility.

We have now also begun to explore mating strategies, especially pair bonding, yet another unique human characteristic among mammals. Speculations about how pair bonding developed from our ancestral roots abound and are open to being quantified, modelled, and analysed. Like the grandmothering models, these investigations will involve PDEs or agent-based models.

Modelling cancer immunotherapy (Project)

Supervisor: A/Prof P. Kim (Carslaw 621; phone 9351-2970)

A next generation approach to treating cancer focuses on cancer immunology, specifically directing a person's immune system to fight tumours. Recent directions in cancer immunotherapy include

- Oncolytic virotherapy: infecting tumours with genetically-engineered viruses that preferentially destroy tumour cells and induce a local anti-tumour immune response,
- Preventative or therapeutic cancer vaccines: stimulating a person's immune system to attack tumour colonies to prevent or hinder tumour development,
- Cytokine therapy: using immunostimulatory cytokines to recruit immune cells and enhance existing anti-tumour immune responses.

These treatments can be used alone or in combination with each other or with other forms of treatment such as chemotherapy. Since immunotherapy often involves immune responses against small tumours, often close to inception, they are highly spatially dependent and often probabilistic. The goal of the will be to develop differential equation and possibly probabilistic agent-based models to understand the tumour-virus-immune dynamics around a small, developing tumour and determine conditions that could lead to effective tumour reduction or complete elimination. The project will involve developing the models and schemes for numerically simulating the ODE and PDE systems, and if possible, performing a stability analysis of the ODE system.

The vortex filament equation and solitons in firenados (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

The vortex filament equation is the relatively simple-to-state PDE for a curve in 3-D

$$\gamma_t = \gamma_x \times \gamma_{xx}.$$

This equation models a thin rotating vortex, like a water sprite, a tornado, or even a firenado. It turns out that this equation is transformable into the Nonlinear Schrödinger (NLS) equation. What this means is that solitons exist (and indeed have even been found) within this equation. Moreover, because the NLS has exact solutions, there is a range of closed solutions to this equation. These closed solutions turn out to be torus knots. This project will explore all of these features, as well as compute, simulate and plot closed form solutions and solitons of thin rotating vortices. Differential Geometry would be a plus for this project as well as familiarity with at least some form of computational software tool, though these are not explicitly necessary.

Chemotaxis in models with zero diffusivity (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

Chemotaxis is the movement of a cell via advection either towards or away from a chemical source. It has been used in many biological models, from slime-moulds to motile bacteria, to roadway construction by humans. Typically linear diffusivity has been studied, but lately models where the diffusivity is allowed to go to zero have become of interest. This project will examine the existence of travelling wave solutions in such models, as well as some elementary stability properties of such solutions.

Stability in a model of herd grazing and chemotaxis (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

This project will examine a model of the formation of a herd of grazing animals. The model will focus on two major factors, how the animal seeks food and how the animals interact with each other. Remarkably, the model shares many properties with another, well studied model, that of so-called bacterial chemotaxis. The aim of this project will be to analyse, both numerically and analytically, such a model, and to understand certain special solutions in the model, called travelling waves, as well as their stability.

Absolute and convective instabilities (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

Absolute and convective instabilities are instabilities that result from points in the essential spectrum crossing into the real axis. This project will focus on several toy problems which explore what happens as such instabilities are present. The aim of this project is twofold - first to explore the mechanisms that lead to such spectral instabilities, and then secondly to understand the dynamic implications of such instabilities.

Absolute spectrum of St. Venant roll waves (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

Roll waves are a phenomenon that occurs when shallow water flows down an inclined ramp. Mathematically they can be modelled by the St. Venant equations. Typically roll waves occur as periodic solutions, however if they are far enough apart, they can be treated as solitary waves. In this case, the spectrum of the linearised operator governs their dynamics, and in particular, their stability properties. This project will focus on computing the absolute and essential spectrum of these solitary waves. Medium computational skills are required for this project.

Perron-Frobenius methods in PDEs (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

The Perron-Frobenius theorem is a cute theorem from linear algebra, which states that a matrix with only positive entries, has a simple largest eigenvalue, and that the eigenvector for this eigenvalue contains only positive entries. Moreover, this is the only eigenvalue whose eigenvector has this property. This theorem can be extended to linear operators, and as such can be used to gain information about the spectral stability of solutions to PDEs. This project will begin by investigating applications of this to simple, scalar PDEs, and will then move up to systems. In particular, investigation of stability in population models which involve an allee effect will be particularly amenable to these methods.

The history of continued fractions (Essay)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

The origin of continued fractions is not well known, though it is traditionally placed at around the time of Euclid's algorithm. By manipulating the Euclidean algorithm, one can derive a simple continued fraction of the rational number $\frac{p}{q}$. Continued fractions gained much interest after John Wallis published his famous identity in 1655

$$\frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times \cdots},$$

and it was observed that the expression could be transformed into a continued fraction. Continued fractions continued to be explored through the 19th century with incredibly prominent mathematicians of the age such as Hermite, Jacobi, Gauss, Stieltjes and Cauchy making contributions. Continued fractions (in some form) are still of interest today in the fields of orthogonal polynomials as well as special functions, and integrable systems. This essay will explore the use and theory behind continued fractions from their 'inception' nearly 2000 years ago up through the current time.

The history of the integral (Essay)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

This essay will follow the history of the integral from the method of exhaustion of the ancient greeks, up through the modern notions of the Lebesgue integral. From here the project can go in a couple of different ways - either into modern notions of stochastic and probabilistic integration, into modern notions of integral transforms, or into computer algorithms for symbolic and numeric integration. Perhaps unsurprisingly, there aren't many books available on this subject, so the student will be expected to find most of their own sources for this project, but the Wikipedia articles 'Integral' and 'History of Calculus' and the references therein are a good place to start. If this project is chosen, some care must be taken to ensure that enough mathematics in included in the essay.

Other possible research topics (Essay or Project)
Supervisor: Dr R. Marangell (Carslaw 720; phone 9351-5795)

Other projects under the supervision of Dr Marangell include topics in the areas of nonlinear standing or travelling waves, topics in the application of geometric and topological methods in dynamical systems and PDEs, and other research topics in the history of mathematics and science in general. Examples of nonlinear standing and travelling waves come from models in a wide range of areas which include mathematical biology, chemistry and physics. More specific examples would be standing/travelling waves in population dynamics, combustion models, and quantum computing, but really there are many, many examples, so please contact Dr Marangell for further details.

PDE models for the distribution of ingested lipids in macrophages in atherosclerotic plaques (Project)

Supervisor: Prof M. R. Myerscough (Carslaw 626; phone 9351-3724)

Atherosclerotic plaques are accumulations of lipid (fat) loaded cells and necrotic (dead) cellular debris in artery walls. They are caused by LDL (which carries 'bad cholesterol') penetrating the blood vessel wall, becoming chemically modified (usually oxidised) and setting off an immune reaction. In response to this immune reaction, macrophages (a type of white blood cell) enter the artery wall and consume the modified LDL. In this way macrophages accumulate lipids and as more LDL and more cells enter the vessel walls the population of cells also grows. Other processes can affect the growth or regression of the plaque, such as cell death, cells leaving the tissue and lipid export from inside cells to HDL (which carries 'good cholesterol' which is good because it's been carried away from the plaque). When atherosclerotic plaques grow very large and rupture they can cause heart attacks and strokes which are one of the two leading causes of death in the developed world. (The other is cancer.)

We have written a partial differential equation model for the accumulation of cells and lipids in plaques. In this model, the number of macrophages in the plaque is a function of both time t and accumulated lipid a. The primary equation is an advection equation with nonlinear source and sink terms, including a term with an integral convolution that models what happens when macrophages phagocytose (=eat) other macrophages that are dead or dying. We have done an analysis of this model at steady state when all the processes (lipid ingestion, macrophages leaving the plaque, the action of HDL) occur at a constant rate. This project will build on this analysis and has the aim of producing numerical solutions to the model when model processes are functions of a, the accumulated lipid inside the cell. This project is particularly suitable for students who are interested in applications of mathematics to biomedical problems, have completed a third year unit on PDEs and have at least some experience in coding in Matlab, C, Python or similar.

PDE models for real and fake macrophages in atherosclerotic plaques (Project)

Supervisors: Dr M. Watson (Carslaw 534), Prof M. R. Myerscough (Carslaw 626; phone 9351-3724)

Atherosclerosis, a major cause of heart attacks and strokes, involves the growth of fatty, cellular plaques in the walls of arteries. In these plaques, the main source of fat (lipid) is LDL, which enters the artery wall from the bloodstream, and the two main cell types are macrophages (a type of immune cell) and smooth muscle cells (SMCs; mural cells native to the artery). Macrophages and SMCs have long been thought to have distinct and well-defined roles: macrophages work to find, consume and remove lipid from the plaque, while SMCs generate a cap of tissue that isolates the (dangerous) content of the plaque from the bloodstream. Recent advancements in fate and lineage tracing of plaque cells have, however, completely changed this paradigm. It is now known that many cells previously thought to be real macrophages are, in fact, SMCs that have left the cap and adopted a macrophage-like phenotype (fake macrophages). Evidence suggests that these SMC-derived macrophages consume and remove lipid less effectively than real macrophages, but little is known about how fake macrophages influence plaque progression.

We have developed a preliminary PDE model for the accumulation of macrophages and lipids in atherosclerotic plaques. We consider two separate macrophage populations (real and fake), where each population is a function of both time t and accumulated lipid a. As the two cell populations compete for (and recycle) the same pool of lipid, the dynamics of these populations are intrinsically coupled. The cell populations are modelled by advection equations with nonlinear source and sink terms, including non-local terms that model jumps in internalised lipid content when a cell divides or consumes a dying cell (efferocytosis). In this project, we will use our preliminary model to investigate how fake macrophages, which behave quite differently from real macrophages, can alter plaque dynamics. Should we find that fake macrophages negatively impact plaque progression, we will then numerically investigate potential treatment strategies to mitigate these effects. This project will be suitable for students who are interested in the application of mathematical modelling to cutting-edge biomedical science, and who have completed third year units on PDEs and/or numerical computation.

Mathematical billiards (Essay or Project)
Supervisor: Dr M. Radnović (Carslaw 633; phone 9351-4543)

Mathematical billiards have been an established topic for research for about one century. They have application in any situation involving collisions and reflections. They are used as a model for the popular game of billiards, and also in laser techniques, the statistical interpretation of the second law of thermodynamics wind-tree model, the dynamics of ideal gas, tri-atomic chemical reactions etc. The field of mathematical billiards is at the cutting edge of mathematics research, and work in the field is highly valued: several Fields Medals were recently awarded for contributions in the area. The research on this project can vary from making computer simulations to more theoretical work. Writing an essay is also available.

Poncelet porisms (Essay)
Supervisor: Dr M. Radnović (Carslaw 633; phone 9351-4543)

Suppose that two conics are given in the plane, together with a closed polygonal line inscribed in one of them and circumscribed about the other one. The Poncelet porism states that then infinitely many such closed polygonal lines exist and all of them with the same number of sides. That statement is one of most beautiful and deepest contributions of the 19th century geometry and has many generalisations and interpretations in various branches of mathematics. In this essay, the student will present rich history and current developments of the Poncelet porism.

Elliptical billiards and their periodic trajectories (Essay or Project)
Supervisor: Dr M. Radnović (Carslaw 633; phone 9351-4543)

We consider billiards in a domain bounded by arcs of several conics belonging to a confocal family. When the boundary of such a billiard does not contain reflex angles, the system turns out to be integrable. Geometrically, the integrability has the following manifestation - for each billiard trajectory, there is a curve, called caustic, which is touching each segment of the trajectory. For elliptical billiards, the caustics are conics from the same confocal family.

Integrability implies that the trajectories sharing the same caustic are either all periodic with the same period or all non-periodic.

On the other hand, if there is at least one reflex angle on the boundary, the integrability will be broken, although the caustics still exist. Such billiards are thus called pseudo-integrable and there may exist trajectories which are non-periodic and periodic with different periods sharing the same caustic.

An essay on this topic would provide a review of classical and modern results related to the elliptical billiards. In a project, the student would explore examples of billiard desks.

Control of boundary-layer flows (Essay or Project)
Supervisor: A/Prof S. Stephen (Carslaw 525; phone 9351-3048)

This project is in the field of hydrodynamic stability of boundary-layer flows where viscous effects are important. The aim is towards understanding more fully the transition process from a laminar flow to a turbulent one.

We will consider rotating flows which are relevant to the flow over a swept wing and to rotor-stator systems in a turbine engine. Experiments show that the boundary layer becomes unstable to stationary or travelling spiral vortices.

The project will investigate the effect of different surface boundary conditions on boundary-layer flows over rotating bodies. Effects such as suction, partial-slip, compliance and wall shape can be modelled. Suction, for example, is used to achieve laminar flow control on swept wings.

The resulting system of governing ordinary differential equations will be solved numerically for the basic flow, determining important values such as the wall shear. The linear stability of these flows to crossflow instabilities will be investigated. These take the form of co-rotating vortices, observed in experiments, and only occur in three-dimensional boundary layers.

The flow for large Reynolds number, corresponding to large values of rotation, will be considered. In this case the boundary layer thickness will be very small so asymptotic methods of solution will be used. Different asymptotic regimes will need to be considered and solutions obtained in each region. Matching the solutions between the regimes and satisfying the boundary conditions will lead to an eigenrelation. Inviscid and viscous instability modes will be considered.

The effect of the surface boundary conditions on the disturbance wave number and wave angle will be determined. This will have applications in possible control of boundary layers as boundaries causing stabilisation of the instabilities could lead to a delay in the transition process from a laminar flow to a turbulent flow.

Inverse problems for partial differential equations: A geometric analysis perspective (Essay or Project)
Supervisor: A/Prof L. Tzou (Carslaw 615; phone 9351-1917)

The analysis of mathematical models arising from various imaging techniques has become a prominent research topic due to its potential applications in many fields including oil exploration and early detection of malignant breast tumor. In many cases one asks the question of whether one can deduce interior information about an object by making measurements on its surface. Mathematically this amounts to recovering the model parameters for a partial differential equation from boundary behavior of its solutions.

This project will explore how these mathematical models behave in various geometric settings. We will see how this applied mathematics problem can motivate us to ask interesting questions in both geometry and analysis.

Pattern formation through geometric microlocal analysis (Essay or Project)
Supervisor: A/Prof L. Tzou (Carslaw 615; phone 9351-1917)

This essay is under the supervisor's ARC Discovery Project *Probing the Earth and the Universe with Microlocal Analysis* and has the potential to continue to a PhD position partially funded by ARC and jointly supervised by applied mathematicians at Macquarie University. There can also be a computational component to the project.

The student will have opportunities to attend, subject to availability, the workshop organized by the supervisor at the Banff International Research Station and Fudan Conference on Microlocal Analysis. During the first semester student will also have opportunity to interact with prominent mathematicians in related fields who are visiting through the School's International Visitor's Program.

We model the effect of drought on vegetation patterns on variable terrains. Mathematically these are described by the dynamics and stability of localized patterns in reaction-diffusion systems on surfaces of non-constant curvature. In systems such as the (rescaled) Schnakenberg model which often serves as a prototype for such studies, the behaviour of localized patterns depend on the asymptotic of the singularity for the Green's function of the Helmholtz operator near the singularity. In the general geometric setting, it is often difficult to derive an explicit formula for the Green's function (or even to compute it numerically), which is why existing theory can only treat simple geometries such as the disk and sphere.

In the language of microlocal analysis, however, this difficulty becomes a standard parametrix construction. This recent breakthrough is allowing us to study pattern formation on terrains which were previously inaccessible.

Non-equilibrium statistical mechanics of a chain: the birth of temperature (Essay or Project)
Supervisor: Dr G. Vasil (Carslaw 627; phone 9351-4163)

If you release a simple, frictionless pendulum from a given height, it will oscillate forever in periodic motion. If you attach a second pendulum at the bottom of the first, the result becomes chaotic with all the associated rich behaviour. But the system will still more-or-less swing back and forth forever.

If you attach 100, 1,000, or 1,000,000 more frictionless elements to the bottom of the configuration something even more interesting happens. The swinging motion begins to go away. The initial coherent kinetic energy transfers into the small-scale random fluctuations of the individual elements. The transfer of energy is a process called "thermalisation" (or the birth of temperature), and the damping of the motion is the emergence of friction in an otherwise dissipation-free system. Even more fascinating, the system can generate configurational entropy, and this gives rise to emergent entropic forces. This is a fancy way of saying that the initial inextensible pendulum chain configuration turns into a hot bouncing bungee cord.

How does this happen exactly? How fast does temperature, entropy, friction, and tension emerge? What can we predict about the statistics? What happens if we drive the system externally? What does this say about hydrodynamical turbulence? This project will investigate all any and all of these question and more. The project will start by directly simulating the pendulum system numerically. After analysing the results, we can use theoretical techniques from equilibrium and non-equilibrium statistical mechanics to better understand these kinds of systems.

Computational projects with Dedalus (Essay or Project)
Supervisor: Dr G. Vasil (Carslaw 627; phone 9351-4163)

Dedalus is a multi-purpose solver than can simulate a broad array of nonlinear partial differential equations (PDEs). You can find out about Dedalus here: http://dedalus-project.org

I have been leading the development of this project for the past few years. The idea behind Dedalus is that the user can enter their equations in text-based format and experiment with different behaviour much more easily than anything that came before. The code runs on computers ranging from laptops to the world's most powerful supercomputers. You can see some examples of past work done here: http://vimeo.com/dedlaus

In the past, my collaborators and I have solved problems ranging from solar magnetohydrodynamics, the dynamics of boats in Arctic seas, buckling elastic beams, nonlinear optics on topologically complex graphs, turbulent thermal convection, nuclear burning flames in white dwarf stars, cancer modelling, and more.

I have a range of potential projects that students can investigate with Dedalus. Some of these are extensions of past problems, and some are entirely new. I'm also happy to talk to students who have a particular application in mind that they would like to try. The idea is to simulate something interesting and model the behaviour after you know the answer!

Improved numerical algorithms for multidimensional spectral methods (Essay or Project)
Supervisor: Dr G. Vasil (Carslaw 627; phone 9351-4163)

Simulating important physical, biological, finical, and mathematical dynamics usually involves solving a large system of linear equations. An important key to many numerical methods is finding a way to solve linear systems in as few operations as possible. The first way to do this is to use a basis that produces matrices with as few non-zero elements as possible. But even with very sparse matrices, then actual time to find the solution can still differ by many orders of magnitude with different methods.

Two general classes of methods exist for solving large, sparse, linear systems. The first class is "direct methods." These are things like Gaussian elimination that you commonly learn in undergraduate courses. The second class are "iterative methods." Iteration involves applying a matrix to a vector over and over again in a clever way that converges to the desired solution. Both of these methods have their pros and cons.

The idea of this project is to study iterative methods in a new context. This project has the potential to accelerate important computational algorithms by many orders of magnitude. The results of this would contribute back to the code base of the Dedalus project and apply to many different areas.

Possible research topics (Essay or Project)
Supervisor: Prof M. Wechselberger (Carslaw 628; phone 9351-3860)

Projects under the supervision of Prof M. Wechselberger include research topics in the areas of relaxation oscillators, return maps, physiological rhythms, mathematical neuroscience, and more generally, dynamical systems.

These areas of research are concerned with the study of oscillatory patterns of so called 'slow/fast systems'. These systems are ubiquitous in nature and control most of our physiological rhythms. For instance, one cycle of a heart beat consists of a long interval of quasi steady state interspersed by a very fast change of state, the beat itself. This very fast relaxation of energy leads to the notion of a relaxation oscillator and shows the appearance of multiple time-scales (slow/fast) in the system.

To study periodic solutions in such systems one can analyse an associated return map. It is well known that already 1D maps can have a rich variety of dynamics, like periodic, quasiperiodic and chaotic solutions, the most famous example is the 1D circle map. Plotting the winding number versus a control parameter in this case yields the famous 'devil's staircase'. Topics could range from a theoretical study of possible dynamics of return maps associated with relaxation oscillators to the analysis of a concrete physiological rhythm.

For more information on possible topics, please see the research link on Prof M. Wechselberger's webpage `http://www.maths.usyd.edu.au/u/wm/`

5 Prizes and Awards

The following prizes may be awarded to Applied Mathematics Honours students of sufficient merit. Students do not need to apply for these prizes, which are awarded automatically. A complete list of the prizes and scholarships offered by the School of Mathematics and Statistics can be found at `http://www.maths.usyd.edu.au/u/About/prizes.html`

Joye Prize in Mathematics — Value: **$5300, with medal and shield**

Awarded to the most outstanding student completing Honours in the School of Mathematics and Statistics.

University Medal

Awarded to Honours students who perform outstandingly. The award is subject to Faculty rules, which require a Faculty mark of 90 or more in Applied Mathematics Honours and a Third Year WAM of 80 or higher. A medal is always awarded when the Faculty mark is 95 or higher. More than one medal may be awarded in any year.

K. E. Bullen Memorial Prize — Value: **$1000**

Awarded on the recommendation of the Head of the School of Mathematics and Statistics in consultation with the professors of Applied Mathematics to the most proficient student in Applied Mathematics Honours, provided that the student's work is of sufficient merit.

Barker Prize — Value: **$550**

Awarded at the Fourth (Honours) Year examination for proficiency in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

M. J. and M. Ashby Prize — Value: **$360**

Offered for the best essay, submitted by a student in the Faculty of Science, that forms part of the requirements of Honours in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

Norbert Quirk Prize No IV — Value: **$250**

Awarded for the best essay on a given mathematical subject by a student enrolled in a Fourth Year course in Mathematics (Pure Mathematics, Applied Mathematics or Mathematical Statistics) provided that the essay is of sufficient merit.

Australian Federation of Graduate Women Prize in Mathematics. — Value: **$175**

Awarded on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate for the degree of BA or BSc who graduates with first class Honours in Applied Mathematics, Pure Mathematics or Mathematical Statistics.

Chris Cannon Prize — Value: **$100**

For the best adjudged essay/project seminar presentation of an Applied Mathematics Honours student.

6 Rights and Responsibilities

Applied Mathematics Honours students will have access to the following.

- Office space and a desk in the Carslaw building.
- A computer account with access to e-mail and the World-Wide Web, as well as TEX and laser printing facilities for the preparation of essays and projects.
- A photocopying account paid by the School for assembling essay/project source material.
- After-hours access key to the Carslaw building. (A deposit is payable.)
- A pigeon-hole in room 728 — please inspect it regularly as lecturers often use it to hand out relevant material.
- Participation in the School's social events.
- Class representative at School meetings.

Applied Mathematics Honours students have the following obligations.

- Regular attendance at the regular weekly seminars in applied mathematics.
- Have regular meetings with project/essay supervisors, and meet all deadlines.
- Utilise all School resources in an ethical manner.
- Contribute towards the academic life in Applied Mathematics at the School of Mathematics and Statistics.

7 Life After Fourth Year

Postgraduate Studies

Many students completing the Honours programme have in the past gone on to pursue postgraduate studies at the University of Sydney, at other Australian universities, and at overseas universities. Please see the Director of Postgraduate Studies if interested in enrolling for a MSc or PhD at the School of Mathematics and Statistics. Students who do well in Applied Mathematics Honours may be eligible for postgraduate scholarships, which provide financial support during subsequent study for higher degrees at Australian universities. The honours coordinator is available to discuss options and provide advice to students interested in pursuing studies at other universities.

Careers

Students seeking assistance with post-grad opportunities and job applications should feel free to ask lecturers most familiar with their work for advice and written references. The Director of the Applied Mathematics Teaching Programme, the Course Coordinator and the unit lecturers may also provide advice and personal references for interested students.