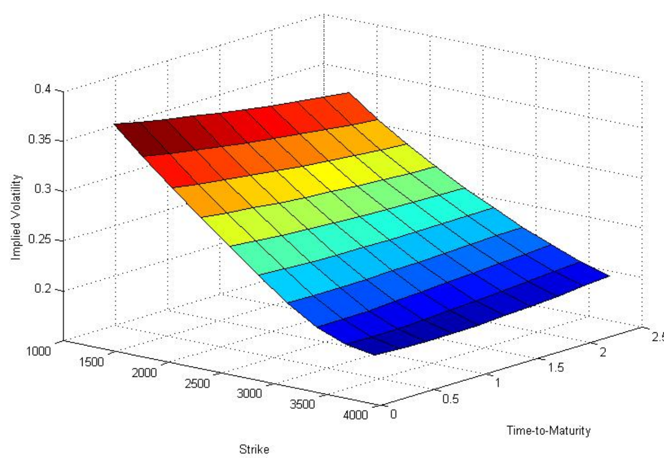
HANDBOOK

Financial Mathematics and Statistics Honours



School of Mathematics and Statistics

The University of Sydney

2020

Financial Mathematics in SMS

Research in Financial Mathematics is interdisciplinary, but it primarily hinges on sophisticated mathematical tools such as: theory of probability, theory of martingales, Ito's stochastic calculus, stochastic differential equations, partial differential equations, optimisation methods, stochastic optimal stopping, stochastic optimal control, stochastic backward differential equations, Dynkin's games, stochastic differential games, statistics of stochastic processes, time series and, last but not least, computational methods for Finance. The School of Mathematics and Statistics offers a wide spectrum of units of study in the area of Financial Mathematics and Statistics, which cover most of the above-mentioned areas of mathematical knowledge and range from introductory units for undergraduates to advanced units for honours students.

We give you an opportunity to complete a high-quality teaching programme capable of competing with analogous programmes offered at other universities in Australia. Financial Mathematics and Statistics studies are designed to meet the need for high level quantitative and modelling skills in the banking, insurance, and finance industries. Our programme will give you a broad introduction to the methods and ideas of Mathematical Finance and will prepare you for employment in the financial sector and for further study in the field.

Our graduates are in very high demand by the finance industry in Australia. Depending on their skills and expertise, they can seek employment as either quantitative analysts (*quants*) or risk managers in a large variety of roles, such as:

- front office/desk quants who work on implementation of pricing and hedging models directly used by traders,
- model validating quants who independently implement pricing models in order to validate models used by the front office models,
- research quants whose task is to invent and develop new pricing approaches and original models for new financial products,
- capital quants who work on modelling the banks credit exposures and capital requirements imposed by a regulatory agency (APRA),
- quant developers who deal with computer programs for implementation of pricing models,
- statistical arbitrage quants who work on finding patterns in market data to support HFT (high-frequency trading) automated trading platform.

Prospective employers for graduates in Financial Mathematics and Statistics can be roughly classified as follows (with some representative instances from Australia)

- major retail banks, e.g., CBA, NAB, ANZ, Westpac, Bank of Queensland,
- investment banks, e.g., Macquarie Group, UBS, Credit Suisse, Goldman Sachs, HSBC,
- hedge funds, e.g., K2 Asset Management, Platinum Asset Management,
- wealth management companies, e.g., AMP, Vanguard Investment, Russell Investments,
- proprietary trading firms, e.g., Optiver Asia Pacific, Propex Derivatives, Lepus,
- accountancy firms, e.g., PricewaterhouseCoopers (PwC), Ernst and Young, Deloitte, KPMG,
- consulting firms, e.g., Accenture, Ernst and Young, Deloitte Consulting, PwC,
- insurance companies and superannuation funds,
- specialised software companies.

Financial Mathematics and Statistics Honours in 2020

Honours year format: coursework 24 cp + project/essay 24 cp

Project: project or essay in Financial Mathematics or Stochastic Analysis = 24 cp

Coursework: 3 core units (18 cp) + 1 selective unit (6 cp) = 24 cp

Progression in learning

	Semester 1		Semester 2
Probability and Stochastic Analysis:	STAT4528	$\Rightarrow$	MATH4512
			$\Downarrow$
Applications in Financial Mathematics:	MATH4511	$\Rightarrow$	MATH4513

Semester 1

The following **core units** are compulsory for all FMS honours students:

STAT4528 Probability and Martingale Theory

MATH4511 Arbitrage Pricing in Continuous Time

Semester 2

At least one of the following **core units** should be completed:

MATH4512 Stochastic Analysis

MATH4513 Topics in Financial Mathematics

Selective 4000-level units

At most one (two upon permission) of the following **selective units** should be completed:

MATH4071 Convex Analysis and Optimal Control (Semester 1)

MATH4411 Applied Computational Mathematics (Semester 1)

MATH4412 Advanced Methods in Applied Mathematics (Semester 2)

STAT4021 Stochastic Processes and Applications (Semester 1)

STAT4025 Time Series (Semester 2)

or any suitable 4000/5000-level unit offered by the School of Mathematics and Statistics.

Exemptions:

A permission for exemption from any unit may be granted provided that the student's application is supported by the supervisor and justified by the nature of the project/essay (for instance, by the clear domination of either computational or statistical aspects of the project).

Further information about project topics:

Please contact Dr Anna Aksamit, Prof Ben Goldys, Prof Marek Rutkowski or Dr Zhou Zhou from the School of Mathematics and Statistics. For their email addresses see:

www.maths.usyd.edu.au/ut/people.html

Entry Requirements to FMS Honours

Research in Financial Mathematics belongs to the realm of Applied Mathematics with the special emphasis on Probability Theory and Stochastic Processes and thus students completing the FMS honours will have a unique opportunity get familiar with a large variety of mathematical and statistical tools used in financial applications.

Since Honours in Financial Mathematics and Statistics (FMS) is a highly specialised research programme, it has different entry requirements and separate core units of study from either Honours in Applied Mathematics or Honours in Statistics.

Notice that the entry requirements vary slightly depending on whether the candidate has completed a Bachelor of Science (BSc) degree or a Bachelor of Advance Studies (BAS) degree. Entry to the Honours Programme in Financial Mathematics and Statistics is usually based on satisfying the following criteria (and subject to approval by the Head of the School of Mathematics and Statistics):

1. **Faculty requirements:**

   BSc: The candidate must have qualified for the Pass Degree in Mathematics (or discipline cognate with Mathematics) with a SciWam of at least 65, or

   BAS: Completed the Bachelor degree with two majors, at least one of which must be in Mathematics (or a discipline cognate with Mathematics) with a WAM of at least 65, and

2. **Mathematics requirements:**

   BSc The candidate must have completed 24 credit points of 3000 level (or 4000 level) either Mathematics or Statistics units of study with an average mark of at least 65% for advanced level units and an average mark of at least 75% for mainstream units.

   BAS The candidate must have completed either the Financial Mathematics and Statistics or Mathematics major with an average mark of at least 65% for advanced level Mathematics or Statistics units and an average mark of at least 75% for mainstream units. In borderline cases, the decision whether a major is cognate with FMS honours programme is in the hands of the FMS honours coordinator and the Faculty.

3. **Project or essay supervision:**

   BSc/BAS The candidate is expected to find a prospective supervisor from among the Applied Mathematics or Statistics staff, who is agreeable to supervise the candidate's essay or project in the candidate's chosen topic.

Students from institutions other than the University of Sydney must possess qualifications which are deemed equivalent to the above. There is some flexibility in these conditions; students not quite meeting them, but keen to pursue Honours in Financial Mathematics and Statistics are invited to contact the Applied Mathematics Honours coordinator for advice. Students are expected to meet with prospective supervisors to discuss the potential for projects or essays before submitting the honours application.

Applications must be submitted to the Faculty of Science.

Application and enrolment information should also be obtained from the Faculty of Science, either in person of from their website. The Faculty will also provide AAM computations once the final third-year results are in.

For further details, please contact the Applied Mathematics Honours coordinator:

Prof Marek Rutkowski: email marek.rutkowski@sydney.edu.au

Core Units of Study

Full-time students normally will enrol in 48 credit points, 24 of which will be course work. Student pursuing honours under the BSc degree will enrol in the 'shell units' MATH4401 A-D, while students in the BAS degree will enrol directly in the units themselves. STAT4528 and MATH4511 are compulsory units; in addition either MATH4512 or MATH4513 (or both) should be taken. In addition to the units of study listed below, students may be allowed to take one course from a related discipline. Approval from the project/essay supervisor and the honours coordinator needs to be obtained prior to enrolment. The choice of course work has to ensure that the student covers the proper material required for the student's project or essay. An assessment scheme for most units is 60% from the final examination and 40% from two assignments.

Units of study in Semester 1: Both units are compulsory for the FMS honours.

STAT4528 Probability and Martingale Theory A/Prof Uri Keich

Probability Theory lays the theoretical foundations that underpin the models we use when analysing phenomena that involve chance. This unit introduces the students to modern probability theory (based on measure theory) that was developed by Andrey Kolmogorov. You will be introduced to the fundamental concept of a measure as a generalisation of the notion of length and Lebesgue integration which is a generalisation of the Riemann integral. This theory provides a powerful unifying structure that brings together both the theory of discrete random variables and the theory of continuous random variables that were introduced earlier in your studies. You will see how measure theory is used to put other important probabilistic ideas into a rigorous mathematical framework. These include various notions of convergence of random variables, zero-one laws, conditional expectation, and the characteristic function. You will then synthesise all these concepts to establish the Central Limit Theorem and to thoroughly study discrete-time martingales. Originally used to model betting strategies, martingales are a powerful generalisation of random walks that allow us to prove fundamental results such as the Strong Law of Large Numbers or analyse problems such as the gambler's ruin. By doing this unit you will become familiar with many of the theoretical building blocks that are required for any in-depth study in probability, stochastic systems or financial mathematics.

MATH4511 Arbitrage Pricing in Continuous Time Dr Zhou Zhou

We explore in this course the theoretical pricing methods, which are widely used by the financial industry to value derivative securities and mitigate risk. We work throughout under the assumption that market prices of financial assets are driven by the Wiener processes. The arbitrage-free prices of options and other derivatives in the ubiquitous Black-Scholes model are obtained using the notion of the martingale measure. Combined with the Ito Lemma, this result leads to a second-order parabolic PDE for the price of any path-independent financial derivative of a European style. By either solving this equation or using the risk-neutral valuation, we obtain, in particular, the celebrated Black-Scholes pricing formula for the call option. Subsequently, we show how to extend their approach to pricing and hedging of foreign market derivatives. We also examine in some detail the pricing and exercising problems for contracts of American style. A large part of this course is devoted to extensions of the Black-Scholes model, specifically, the CEV model, the local volatility model calibrated to market data, and stochastic volatility model. The course concludes by an analysis of the issue of robustness of the Black-Scholes approach.

Units of study in Semester 2: At least one of these units should be taken.

MATH4512 Stochastic Analysis Prof Marek Rutkowski

Capturing random phenomena is a challenging problem in many disciplines from biology, chemistry and physics through engineering to economics and finance. There is a wide spectrum of problems in these fields, which are described using randomly evolving processes. Hence it is of crucial importance to equip an applied mathematician with tools used to analyse and quantify random phenomena. Modern theory of financial markets relies on advanced mathematical and statistical methods that are used to model, forecast and manage risk in complex financial transactions. After the publication of the ground-breaking paper of Black and Scholes (1973) on arbitrage pricing of European options, it became clear that Stochastic Analysis and the theory of martingales are indispensable tools for the theory of arbitrage-free financial markets, derivation of prices of standard options and other derivative securities, and hedging of financial risks. In this unit, you will get familiar with the basic concepts and techniques of Stochastic Analysis, such as: the Brownian motion, continuous (local) martingales, the Itô stochastic integral, the Itô formula, stochastic differential equations, stochastic exponential, equivalent change of a probability measure and the Girsanov theorem, integral representation of martingales with respect to a Brownian filtration, relations to second order partial differential equations, and the Feynman-Kac formula. By completing this unit, you will gain a deep knowledge about stochastic integration, which is an indispensable tool for further studying

MATH4513 Topics in Financial Mathematics Prof Marek Rutkowski

The fixed-income market is the sector of the global financial market on which various interest rate- sensitive instruments, such as: bonds, swaps, swaptions and caps, are traded. In practice, several fixed income markets operate and thus many concepts of interest rates have been developed. The pricing and hedging of interest rate derivatives is an important and complex issue, which creates a demand for sophisticated stochastic models capable of dealing with all kinds of interest rate risks. Theoretical term structure models are often formulated in terms of interest rates that are different from the conventional market rates. The first goal of this unit is to study various kinds of interest rates, such as: the short-term rate, the instantaneous forward rates, LIBORs and swap rates. We build stochastic models for theoretical and quoted interest rates and we examine mathematical techniques used to value and hedge fixed income securities and related derivatives. The second part of this unit is devoted to mathematical tools used for the valuation and hedging of defaultable claims, such as: vulnerable options and corporate bonds, and various classes of credit derivatives, such as: credit default swaps (CDSs) and collateralised debt obligations (CDOs). We first examine the main developments within the so-called structural approach to modelling and valuation of credit risk. In particular, we analyse Merton's model of corporate debt and the first-passage-time approach due to Black and Cox. Next, we study the so-called hazard rate approach to modelling of credit events and dependent default times, and credit ratings. Subsequently, we examine hedging strategies for multi-name credit derivatives, such as first-to-default swaps, under the assumption that some credit-risky securities (such as, e.g., corporate bonds or single-name credit default swaps) are traded. We conclude by a brief introduction to the so-called valuation adjustments through solutions to nonlinear BSDEs.

Assessment Procedures

The Honours mark for each student is computed based on the following:

- 50% for the Project/Essay assessment;
- 50% for 4 units of study (12.5 % for each).

The marking scale for Honours is significantly different from the undergraduate marking scale at the University of Sydney. The Essay/Project, in addition to all the fourth-year units of study, will be marked with this scale in mind. This scale appears below.

Grade of Honours	Faculty Scale
First Class, with Medal	95–100
First Class (possibly with Medal)	90–94
First Class	80–89
Second Class, First Division	75–79
Second Class, Second Division	70–74
Third Class	65–69
Fail	00–64

All assessable student work (such as assignments, essays and projects) should be completed and submitted by the advertised date. If this is not possible, approval for an extension should be sought *in advance* from the lecturer concerned or, in the case of essays and projects, from the honours coordinator. Unless there are compelling circumstances, and approval for an extension has been obtained in advance, late submissions will attract penalties as determined by the Board of Examiners (taking into account any applications for special consideration).

Appeals against the assessment of any component of the course, or against the class of Honours awarded, should be directed to the Head of School.

The Project/Essay

A significant part of the Honours year is the completion of an Honours Project or Essay by each student. There is a distinction between a project and an essay. A project involves intensive research, analysis or computation and normally requires a greater level of supervision than an essay. An essay may cover a classical problem of acknowledged importance and mathematical depth with the student providing his/her own critical evaluation.

Each student must choose an project/essay supervisor who is willing to supervise the student's chosen topic for the project or essay. The supervisor must be a member of the Applied Mathematics staff of the School of Mathematics and Statistics. However, student are welcome to choose different topics, provided that they are able to obtain a supervisor for that topic from within the School. Project/essay topics and supervisors should be finalised by the beginning of Semester 1, so that students can commence work immediately on their projects/essays.

Assessment of the Project/Essay

The project or essay will be marked according to the following.

- **90% for the final written report**

  This will be marked by 3 different markers, one of whom is the supervisor, and each marking will therefore constitute 30% of the final project/essay mark. Note that the assessment also includes a one page report submitted at the end of the first semester.

- **10% for a seminar presentation on the project/essay**

Three typed and bound copies of the final project/essay should be submitted to the Applied Mathematics Honours coordinator, who will then distribute these copies to three markers (one of which is the supervisor) for marking.

The seminar is an opportunity for each student to present the material of his/her project/essay to a mathematically literate audience. The seminar talk will usually be of 25 minutes duration, with an additional 5 minutes set aside for questions. The honours coordinator will provide additional information and help students in their preparation for the seminar. The presenter of the best Applied Mathematics honours seminar will be awarded the Chris Cannon Prize.

Important Dates

The following are important dates for all students intending to complete their projects/essays by the end of the first or the second semester of 2020.

Semester 1:

- **Seminar: Week of April 20-24th, 2020 (week 8)**

- **Essay Submission: Monday, 25th May 2020 (week 13)**

  An electronic file (pdf format) and three typed and bound copies of the project/essay are to be handed in to the Applied Mathematics Honours coordinator by this date and time. Note that the electronic pdf file must be submitted to the honours coordinator and uploaded via the Turnitin system to the University's LMS website by this deadline.

Semester 2:

- **Seminar: Week of 21-25 September, 2020 (week 8)**

- **Essay Submission: Monday, 2nd November 2020 (week 13)**

  An electronic file (pdf format) and three typed and bound copies of the project/essay are to be handed in to the Applied Mathematics Honours coordinator by this date and time. Note that the electronic pdf file must be submitted to the honours coordinator and uploaded via the Turnitin system to the University's LMS website by this deadline.

Project/Essay Guidelines

- The student should consult the supervisor on a regular basis, preferably at least once a week. This is the student's responsibility.
- A realistic schedule for work on the project or essay should be drawn up at an early stage, and adhered to as closely as possible. If it proves necessary to modify the original plans, a revised schedule should be drawn up after discussion with the supervisor.
- At the end of Semester 1, a one page report has to be submitted to the honours coordinator. This report includes a half page description about the students aim/scope of the project/essay and a half page description about what the student has achieved in Semester 1 and what the student wants to achieve in Semester 2. This report has to be approved by the supervisor before submission.
- The project/essay should be both a discursive and a critical account of the selected topic. The work must contain substantial mathematical content.
- The project/essay should be based on some four to six original primary source articles, which themselves represent a substantial contribution to the topic. Secondary sources, such as books, review papers, etc., should also be consulted and cited.
- Original research is not essential for essays.
- The length of the project/essay should be between 40 to 60 typed A4 pages. Only in exceptional circumstances, and after consultation with the supervisor, should the essay exceed 60 pages. This number includes all figures, contents pages, tables, appendices, etc. Computer programs essential to the work should be included (with adequate commentary) as additional material.
- Students should be careful to provide full and correct referencing to all material used in the preparation of projects and essays. Be explicit in stating what is your contribution and what is someone else's contribution. Avoid quoting verbatim unless you wish to reinforce an important point.
- Three examiners will be appointed to assess each project/essay. One of these examiners will be the student's supervisor. Although marking schemes may differ, marks will generally awarded for:

  (i) selection and synthesis of source material;
  (ii) evidence of understanding;
  (iii) evidence of critical ability;
  (iv) clarity, style and presentation;
  (v) mathematical and/or modelling expertise.

- Students are advised to read the pamphlet entitled "*Guide to Essay Writing for Science Students*" available from the Science Faculty Office.
- The preferred method of typesetting mathematical documents these days is using LaTeX. This is available from the computers at the School. Students are recommended to use LaTeX in typesetting their Essays/Projects.
- Students who have worked on their essays or project topics as Vacation Scholars are required to make a declaration to that effect in the preface of their project/essay.

Topics of FMS Honours Projects/Essays

You will find below a list of possible project/essay topics for honours students in Financial Mathematics and Statistics. Prospective students interested in any of these topics are encouraged to discuss them with the named supervisors as early as possible. Let us stress this list is not exhaustive and thus you may wish to suggest your own topic for essay or project or discuss any other topic with a potential supervisor. Before commencing work, however, each student must find a member of staff who will agree to supervise the project/essay. For topics other than those listed below, the student and supervisor must submit a brief written outline of the proposed topic for approval by the FMS honours coordinator.

Asymmetric information in modelling of financial markets Dr Anna Aksamit

Differing levels of information available to agents are an essential aspect of decision-making problems. It is natural to conjecture that an agent who possesses more information in a financial market typically gains advantage over less-informed agents. In this project, you will investigate mathematical framework used to formalise the idea of different levels of information available to agents in a stochastic model of a financial market and quantify the role of information in various decision-making problems. The essential tool to address these problems is the enlargement of filtration theory, which is an actively researched area of stochastic analysis. You will learn the recent results from that theory and you will apply them to solve utility maximisation problems incorporating two distinct information flows. The related literature includes the recent monograph by Aksamit and Jeanblanc (2017) and the paper by Chau et al. (2018).

Optimisation problems for stochastic stopping games Dr Anna Aksamit

Game theory, which focuses on solutions to optimisation problems with interactions between agents, has undoubtedly plenty of applications in various fields, in particular, economics and finance. In this project, you will study a particular class of two-player zero-sum stopping games, which are also known as Dynkin games, where the players choose their respective optimal stopping rules. The main theoretical goal of the project/essay is to investigate the most essential notions with optimal stopping rules, such as the value of the game (notice that even the existence of the value of the game is by no means ensured a priori) and the classical concept of the Nash equilibrium. In the next step, it is also possible to analyse applications to arbitrage-free pricing of some classes of game options. You may pursue to investigate several variations of the game, including asymmetric information or the case of agents with heterogeneous beliefs. The related literature includes the recent papers by Ekström et al. (2017), Esmaeeli et al. (2018), Gensbittel and Rainer (2018) and Grün (2013).

Exploration and exploitation in reinforcement learning Dr Anna Aksamit and Dr Zhou Zhou

The project/essay focuses on stochastic control problem where an agent is choosing the best action to maximize their reward in random environment. Instead of classical setting where parameters of a random model are fully known, you will study the reinforcement learning setting where the underlying model is unknown and hence dynamic learning is needed, the agent employs exploration to interact with and learn the unknown environment through trial and error. There is a trade-off between exploration of a black box environment and exploitation of current knowledge. The agent must balance between greedily exploiting what has been learned to choose actions, and continuously exploring the environment to acquire more information to potentially achieve long-term benefits. This project will be based on the papers *Exploration versus exploitation in reinforcement learning: A stochastic control approach* and *Continuous-time mean-variance portfolio selection: A reinforcement learning framework* by Wang et al. (2019).

Pricing of Australian fixed income securities Dr Igor Geninson and Prof Marek Rutkowski

In the first part of the project, the goal is to build and calibrate yield curves for Australian government, state and corporate bonds and FRNs using Nelson-Siegel-Svensson model. The calibration should be performed using alternative optimisation techniques like the DE (differential evolution) algorithm or a similar method; see M. Gilli, S. Grosse and E. Schumann: *Calibrating the Nelson-Siegel-Svensson model* (https://ssrn.com/abstract=1676747) or A. Posthaus: *Yield curve fitting with artificial intelligence: a comparison of standard fitting methods with artificial intelligence algorithms* (https://ssrn.com/abstract=3359599).

Part two of the project is concerned with building a set of single issuer curves for the Australian fixed income securities and calibration of the single issuer curves to observed market prices. The final stage of the project hinges on incorporating additional available information, e.g. credit default products, into the single issuer curves, using robust finance methods, as developed, for instance, in the paper by A. Aksamit, Z. Hou and J. Obłój: *Robust framework for quantifying the value of information in pricing and hedging*, 2016.

Stochastic PDEs in credit modelling Prof Ben Goldys

Pricing and hedging of credit derivatives is one of the most challenging problems of Mathematical Finance. A standard tool to build mathematical models in this area is the theory of copula functions. A major deficiency of this method is that it is difficult to incorporate dynamics in the model. This problem has been recently addressed by using the mean-field method well known in Physics in the paper by N. Bush et al.: *Stochastic evolution equations in portfolio credit modelling,* SIAM Journal on Financial Mathematics 2 (2011), 627–664. Using this approach we can derive a stochastic partial differential equation that describes time evolution of the fraction $f(t,x)$ of agents that did not default before a given time and are in a distance x to the default. The aim of this project is to study this approach and develop more realistic models.

SDEs with memory and applications to stochastic volatility models Prof Ben Goldys

A standard assumption of the Black-Scholes model for pricing of options is that the volatility of the asset, if random, is driven by Wiener process. However, recent careful analysis of market data show that the volatility is a much rougher process than the Wiener process (see, for example, J. Gatheral, T. Jaisson, M. Rosenbaum: *Volatility is rough*, https://arxiv.org/abs/1410.3394). In this project we will investigate a simple model of stock prices in the case when volatility is modelled as a solution to a stochastic differential equation (SDE) with memory. The main question will be how to derive (or approximate) prices and hedges for financial derivatives.

Designing optimal contracts Prof Ben Goldys

Contract theory is part of economics focused on designing optimal contracts between principals and agents. Only recently, mathematical methods have become available in this area. The breakthrough has been made by Y. Sannikov in his seminal paper *A continuous-time version of the principal-agent problem*, Review of Economic Studies 75 (2008), 957–984. The principal-agent problem (also known as the agency problem) occurs when one person or entity (the *agent*) is able to make decisions and/or take actions on behalf of another person (the *principal*). The dilemma exists in circumstances where agents are motivated to act in their own best interests, which are contrary to those of their principals. The proposed solution to the principal-agent problem is to ensure the provision of appropriate incentives so agents act in the way principals wish. We will study Sannikov's paper and will try to extend its conclusions to more realistic situations. Notice that familiarity with stochastic analysis is necessary for this project.

Representations of American options via European claims Prof Marek Rutkowski

A challenging mathematical problem of great importance for the Finance industry was formulated by Jourdain and Martini (2001): they were interested in pricing of path-independent European claims in the Black-Scholes model and they discovered that, for a large class of European payoffs, in the region where the European price increases with the time to maturity, this price is equal to the American price of another claim. They also furnished particular examples where the corresponding American claims can be computed explicitly. However, the full characterisation of American claims obtained in this way was, and still remains, an open question, although a first step towards verifying the representability of certain American claims was recently done by Lenga (2017). This project/essay combines an intriguing theoretical problem with numerical aspects of arbitrage-free pricing of American options and thus it is particularly suitable for a student with a working knowledge of computational methods.

Analytical valuation under funding costs and credit risk Prof Marek Rutkowski

The credit valuation adjustment (CVA) and the funding valuation adjustment (FVA) are the price adjustments due to the default risk and the cost of funding the trade. Trading desks back the deal with a client by hedging it with other dealers in the market, and this may involve maintaining a number of hedging accounts in the underlying assets, in cash, or in other correlated assets when proxy hedging. The funds needed for these operations are raised from the internal treasury of the dealer and, ultimately, they come from external funders. Interest charges on all borrowing and lending activities need to be covered and this affects the contract's valuation. The causes of these adjustments are accounted for at the level of the contract payoffs and the resulting all-inclusive price is written as a solution to an advanced mathematical problems, such as semilinear PDEs or BSDEs (see Bielecki et al. (2018)). The challenging theoretical goal is to identify classes of simple contracts and market models where this all-inclusive price of an uncollateralized contract can be computed analytically yielding a closed-form expression. Examples of such contracts are given in the existing literature; see, for instance, Brigo et al. (2017) or Bichuch et al. (2018).

Modeling of risky interest curves Prof Marek Rutkowski

The recent credit crunch has changed the markets perceptions and banks are nowadays more conservative about the possibility of default of other banks and their own future funding costs, that is, the rate at which they can borrow. These risk are being priced into money market lending rates (e.g. LIBOR) as well as derivatives written on these rates. The project aims at understanding apparent anomalies that appeared recently in the interest rate curves. Mathematical goal is to develop a model reflecting the fact that the LIBORs embed options on the creditworthiness of the counterparty and to show how this model can be used to explain the basis swaps patterns during the financial crisis by taking as inputs the level of counterparty risk and credit volatility.

Pricing and exercising of Bermudan swaptions Prof Marek Rutkowski

The price of a vanilla Bermudan swaption can be decomposed into the sum of the value of the most expensive component swaptions and the value of an option to switch from one swaption to another under suitable circumstances. It is sometimes argued that matching the market prices of Bermudans requires discounting the value of the switch option at a higher interest rate than the European swaptions. The simplest way to price Bermudan swaptions and calibrate a model to market data is to introduce an additional unknown parameter, which is similar to a tax. This somewhat contrived pricing method was introduced by Hagan (2015) and later modified by Brace (2016). Their approach can be extended to other callable exotics by using suitable adjustments to handle embedded options. An important question is whether a self-financing hedging portfolio can be constructed to match the values obtained in a model with artificially added taxation. The goal of the project is to re-examine the pricing approaches to Bermudan swaptions put forward by Hagan (2015) and Brace (2016) and to propose an alternative approach based on the sub-optimality of the holder's exercise policy.

Pricing of superannuation guaranteed benefits Prof Marek Rutkowski

Superannuation guaranteed benefits resemble in some respects American and game options, but their structure is even more convoluted since the holder can make decisions about withdrawals. The goals of the project differ from the classical portfolio optimisation since superannuation guarantee products have several non-standard features: a specific design of the income stream and gains from market upside, in some instances, optionality of withdrawals, a possibility of termination by the holder and, finally, death benefits for designated beneficiaries. In contrast, classical portfolio optimisation problems focus on either achieving the desired level of expected rate of return on a dynamic portfolio of assets or maximisation of the expected utility from the terminal wealth. The superannuation guarantee product need to be valued upfront and then hedged by the insurance provider using highly sophisticated trading strategies to avoid losses in the event of market downturn whereas the standard portfolio optimisation assumes that the initial wealth is given. For the existing literature, see Forsyth and Vetzal (2014), Huang, Zeng and Kwok (2017) and Luo and Shevchenko (2017).

Portfolio optimisation with liquid American options Dr Zhou Zhou

As a classical problem in mathematical finance, portfolio optimisation has been studied extensively in various setups. American-style options, with huge trading volumes, are perhaps the most popular financial derivatives in the financial market. However, so far there have been very limited studies on portfolio optimisation involving liquidly traded American options. Among the few works, Bayraktar and Zhou (2016) analyses a general utility maximisation problem with the short-selling constraint on liquid American options. They establish a duality result for the value function associated with the utility maximisation problem. In this project/essay, you will consider the portfolio optimisation in which stocks are traded dynamically and American options are traded statically. Different from Bayraktar and Zhou (2016), here we assume that American options can be sold. Due to the nature of American options, investors who short the options will face the uncertainty of the exercise times. How to deal with this uncertainty and what are the corresponding trading strategies will be the core questions for this project/essay. In addition to probability, stochastic processes and control, the project will also involve game theory. The project is particularly suitable for students who are interested in game theory and/or its applications. For some relevant works, see Bayraktar and Zhou (2014, 2017).

Mean-field stopping games Dr Zhou Zhou

Companies or individuals often need to decide the best times (stopping strategies) to take certain actions in order to maximise their profits/utilities. When there is more than one investor (player) involved, players will interact with each other. In this situation, how the players choose stopping strategies is of particular interest (e.g., competing companies choose times to enter/quit the market or update products). Such real-world problems can be modelled as stopping games. There have been a lot of works on stopping games with different applications. Traditional methods of game theory assume that each player keeps an eye on every other player. As a result, roughly speaking, the dynamics of an n-player game can be represented by a system of (at least) n coupled differential equations. Unfortunately, when n is relatively large, the system of n equations are usually computationally intractable. As can be seen from Zhou (2015), the construction of a three-player stopping game is already very complicated. In practice, when there are a lot of players, each player may not keep track of every other player's behaviour. More likely, she may only pay attention to the aggregate behaviour of the cohort of other players. The above intractability and practical situation motivates us to seek an effective tool to study stopping games with a large number of players, where the payoff of each player depends on her own strategy and relevant features of the aggregate behaviour of other players. The tool of mean-field games serves this purpose. So far, there are only a few works on mean-field stopping games (see, e.g., Carmona et al. (2017), Nutz (2018), Nutz et al. (2018)), and a lot of further research needs to be done in order to deepen the understanding of this topic. In this project/essay, you will consider mean-field stopping games and their applications in finance and economics. You will also investigate some particular models (e.g., bank-run models) and examine the existence of equilibria.

Prizes and Awards

- The following prizes may be awarded to students of sufficient merit.
- Students do not need to apply for these prizes, which are awarded automatically.

Joye Prize in Mathematics Value: **$5300, with medal and shield**

Awarded to the most outstanding student completing Honours in the School of Mathematics and Statistics.

University Medal

Awarded to honours students who perform outstandingly. The award is subject to Faculty rules, which require a Faculty mark of 90 or more in Applied Mathematics 4 and a Third Year WAM of 80 or higher. A medal is always awarded when the Faculty mark is 95 or higher. More than one medal may be awarded in any year.

K. E. Bullen Memorial Prize Value: **$1000**

Awarded annually on the recommendation of the Head of the School of Mathematics and Statistics in consultation with the professors of Applied Mathematics to the most proficient student in Applied Mathematics 4, provided that the student's work is of sufficient merit.

Barker Prize Value: **$550**

Awarded at the Fourth (Honours) Year examination for proficiency in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

M. J. and M. Ashby Prize Value: **$360**

Offered annually for the best essay, submitted by a student in the Faculty of Science, that forms part of the requirements of Pure Mathematics 4, Applied Mathematics 4 or Mathematical Statistics 4.

Norbert Quirk Prize No IV Value: **$250**

Awarded annually for the best essay on a given mathematical subject by a student enrolled in a Fourth Year course in mathematics (Pure Mathematics, Applied Mathematics or Mathematical Statistics) provided that the essay is of sufficient merit.

Australian Federation of Graduate Women Prize in Mathematics. Value: **$175**

Awarded annually, on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate for the degree of BA or BSc who graduates with first class Honours in Applied Mathematics, Pure Mathematics or Mathematical Statistics.

Chris Cannon Prize Value: **$100**

For the best adjudged project/essay seminar presentation of an Applied Mathematics 4 student.

Rights and Responsibilities

Honours students will have access to the following.

- Office space and a desk in the Carslaw building.
- A computer account with access to e-mail and the World Wide Web, as well as $\TeX$ and laser printing facilities for the preparation of projects and essays.
- A photocopying account paid by the School for assembling project/essay source material.
- After-hours access key to the Carslaw building. (A deposit is payable.)
- A pigeon-hole in room 728 – please inspect it regularly as lecturers often use it to hand out relevant material.
- Participation in the School's social events.
- Class representative at School meetings.

Honours students have the following obligations.

- Have regular meetings with project/essay supervisors and meet all deadlines.
- Regular attendance at the weekly seminars *Stochastics and Finance*.
- Contribute towards the academic life in Financial Mathematics at the School of Mathematics and Statistics.
- Utilise all School resources in an ethical manner.