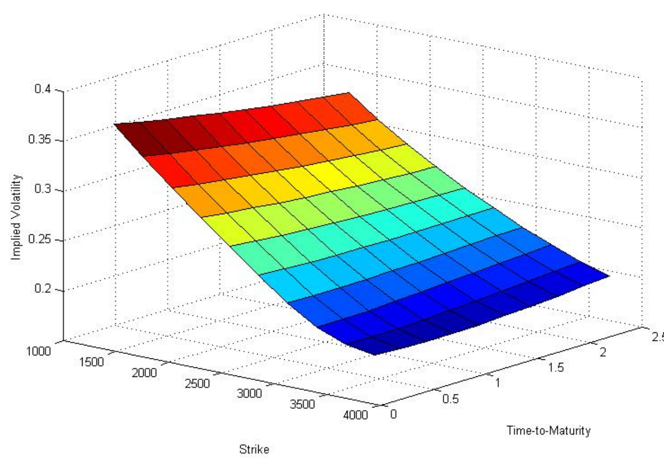
Financial Mathematics and Statistics Honours

Student Handbook 2022



School of Mathematics and Statistics

The University of Sydney

General Information

Honours in Financial Mathematics and Statistics is a one-year program consisting of four 6 credit point coursework units and 24 credit points of project. For more details about the honours program in Financial Mathematics and Statistics, see Table A.

1. **Entry requirements to honours programs**

   Honours is available to students who have a completed major in an area relevant to their project and have met the entry requirement to honours programs in the Faculty of Science. Notice that entry requirements to the honours program in Financial Mathematics and Statistics vary slightly depending on whether the candidate is completing a Bachelor of Science (BSc) degree or a Bachelor of Advanced Studies (BAS) degree. Therefore, it is necessary to check whether you satisfy all entry requirements before applying for admission to honours in Financial Mathematics and Statistics. See also more detailed guidelines for students at the University of Sydney willing to complete an honours year. The most important entry requirements are summarised below but you should consult the website of the Faculty of Science for further details.

   For instance, admission to the Bachelor of Advanced Studies (Honours) requires:

   – completion of a bachelors degree from the University of Sydney or equivalent qualification from another tertiary institution in time to commence honours in the semester for which entry is sought;
   – completion of two majors or study deemed by the supervising faculty to be of comparable depth, of which at least one major is cognate to the honours area for which entry is sought;
   – a final Weighted Average Mark (WAM) of at least 65.00;
   – demonstrated evidence of contact with the School of Mathematics and Statistics, such as email correspondence with a prospective supervisor of a research project and the Honours Coordinator;
   – Expression of Interest (EOI) form signed-off by the Honours Coordinator.

   To qualify for admission to the Bachelor of Science (Honours), you must:

   – have qualified for or be a graduate with a Bachelor of Science degree from the University of Sydney or equivalent qualification from another tertiary institution;
   – have completed a relevant major (i.e. minimum of 24 credit points of 3000-level units of study) relating to the intended Honours discipline; have achieved a Science Weighted Average Mark (SCIWAM is the average over all second and third year units attempted) of at least 65.00 or have a credit average (65.00) in 48 credit points of relevant 2000-level and 3000-level units of study;
   – satisfy any additional criteria set by the relevant Head of School or Discipline;
   – demonstrate evidence of contact with the School of Mathematics and Statistics, such as email correspondence with a prospective supervisor of a research project and the Honours Coordinator;
   – submit Expression of Interest (EOI) form signed-off by the Honours Coordinator.

2. **Guidelines for students at the University of Sydney**

   - If you are currently enrolled in the Bachelor of Science and commenced your studies in 2018 or later (or transferred to the new curriculum version of your degree in 2018), how you commence honours will depend on how much of your degree you have completed.
   - If you have completed 96 credit points or less, we recommend you change to the combined Bachelor of Advanced Studies. You will then be able to undertake honours as an additional fourth year and graduate with two bachelor's degrees.
   - If you have completed 97–120 credit points and are on track to complete two majors, you have the choice to change to the combined Bachelor of Advanced Studies to complete honours or complete your Bachelor of Science then apply for the stand-alone Bachelor of Advanced Studies to undertake honours. You will need to apply before your penultimate semester.
   - If you have completed 121 credit points or more and are on track to complete two majors, you will need to complete your Bachelor of Science and apply for the stand-alone Bachelor of Advanced Studies to undertake honours. You will need to apply before your penultimate semester.
   - An appended honours degree, the Bachelor of Science (Honours), is an option for students who commenced their studies prior to 2018 or who have not completed or are not on track to complete two majors in their single bachelors degree.

3. **Project supervision**

   The candidate is required to find a prospective supervisor from among the School of Mathematics and Statistics staff, who is agreeable to supervise the candidate's project in the candidate's chosen topic. Students are required to submit the Expression of Interest form to the Honours Coordinator before submitting the honours application to the Faculty of Science.

4. **Students from other institutions**

   Students from institutions other than the University of Sydney must possess qualifications which are deemed equivalent to the above.

5. **Online applications to the Faculty of Science**

   Application and enrolment information should be obtained from the Faculty of Science from their website. For online applications to honours programs, see here.

6. **Scholarships**

   For scholarships available to honours students, see the website.

For further details, you may contact the FMS honours coordinator

Prof Marek Rutkowski: marek.rutkowski@sydney.edu.au

Financial Mathematics

Research in Financial Mathematics primarily hinges on sophisticated mathematical tools such as: theory of probability, theory of martingales, Ito's stochastic calculus, stochastic differential equations, partial differential equations, optimisation methods, stochastic optimal stopping, stochastic optimal control, stochastic backward differential equations, Dynkin games, stochastic differential games, statistics of stochastic processes, time series and computational methods. We offer a wide spectrum of units of study in the area of Financial Mathematics and Statistics, which cover most of the above-mentioned areas of mathematical knowledge and range from introductory units for undergraduates to advanced units for honours students. We give you an opportunity to complete a high-quality teaching program capable of competing with analogous programs offered at other universities in Australia. Financial Mathematics and Statistics studies are designed to meet the need for high level quantitative and modelling skills in the banking and finance industries.

Our graduates are in very high demand by the finance industry in Australia and they can seek employment in a large variety of roles, such as:

- front office/desk quants who work on implementation of pricing and hedging models directly used by traders,
- model validating quants who independently implement pricing models in order to validate models used by the front office models,
- research quants whose task is to invent and develop new pricing approaches and original models for new financial products,
- capital quants who work on modelling the banks credit exposures and capital requirements imposed by a regulatory agency, such as APRA,
- quant developers who deal with computer programs for implementation of pricing models,
- statistical arbitrage quants who work on finding patterns in market data to support HFT (high-frequency trading) automated trading platform.

Prospective employers for our graduates:

- major retail banks, e.g., CBA, NAB, ANZ, Westpac, Bank of Queensland,
- investment banks, e.g., Macquarie Group, UBS, Credit Suisse, Goldman Sachs, HSBC,
- hedge funds, e.g., K2 Asset Management, Platinum Asset Management,
- wealth management companies, e.g., AMP, Vanguard Investment, Russell Investments,
- proprietary trading firms, e.g., Optiver Asia Pacific, Propex Derivatives, Lepus,
- accountancy firms, e.g., PricewaterhouseCoopers, Ernst and Young, Deloitte, KPMG,
- consulting firms, e.g., Accenture, Ernst and Young, Deloitte Consulting,
- insurance companies and superannuation funds,
- specialised software companies.

Financial Mathematics and Statistics Honours

Honours year format: research project 24 cp + coursework 24 cp

Project: Research project FMAT4103–4106 = 24 cp

Coursework: 3 core units (18 cp) + 1 selective unit (6 cp) = 24 cp

Progression in learning	Semester 1		Semester 2
Probability and Stochastic Analysis:	STAT4528	⇒	MATH4512
			⇓
Applications in Financial Mathematics:	MATH4511	⇒	MATH4513

Semester 1

The following **core units** are compulsory for all FMS honours students:

STAT4528 Probability and Martingale Theory

MATH4511 Arbitrage Pricing in Continuous Time

Semester 2

At least one of the following **core units** should be completed:

MATH4512 Stochastic Analysis

MATH4513 Topics in Financial Mathematics

Selective units of study

In addition to the core units, students should choose their selective unit from units offered by the School of Mathematics and Statistics listed as 4000 level or higher, which have not already been taken for credit with the proviso that at most one unit labelled 5000 or higher may be taken. For the full list of selective units available to students enrolled in FMS honours, see Table A.

Research project

Each student is required to complete the project unit FMAT4103–4106, which is composed of a written thesis and a seminar presentation. Students should enrol in two project units in each semester: FMAT4103–4104 in Semester 1 and FMAT4105–4106 in Semester 2.

FMS honours coordinator in 2022

Prof Marek Rutkowski
Room 814, Carslaw Building
Email: marek.rutkowski@sydney.edu.au

Core Units of Study

Units of study in Semester 1: Both units are compulsory for the FMS honours.

STAT4528 Probability and Martingale Theory Dr Anna Aksamit and Prof Ben Goldys

Probability Theory lays the theoretical foundations that underpin the models we use when analysing phenomena that involve chance. This unit introduces the students to modern probability theory (based on measure theory) that was developed by Andrey Kolmogorov. You will be introduced to the fundamental concept of a measure as a generalisation of the notion of length and Lebesgue integration which is a generalisation of the Riemann integral. This theory provides a powerful unifying structure that brings together both the theory of discrete random variables and the theory of continuous random variables that were introduced earlier in your studies. You will see how measure theory is used to put other important probabilistic ideas into a rigorous mathematical framework. These include various notions of convergence of random variables, zero-one laws, conditional expectation, and the characteristic function. You will then synthesise all these concepts to establish the Central Limit Theorem and to thoroughly study discrete-time martingales. Originally used to model betting strategies, martingales are a powerful generalisation of random walks that allow us to prove fundamental results such as the Strong Law of Large Numbers or analyse problems such as the gambler's ruin. By doing this unit you will become familiar with many of the theoretical building blocks that are required for any in-depth study in probability, stochastic systems or financial mathematics.

MATH4511 Arbitrage Pricing in Continuous Time Dr Zhou Zhou

We explore in this course the theoretical pricing methods, which are widely used by the financial industry to value derivative securities and mitigate risk. We work throughout under the assumption that market prices of financial assets are driven by the Wiener processes. The arbitrage-free prices of options and other derivatives in the ubiquitous Black-Scholes model are obtained using the notion of the martingale measure. Combined with the Ito Lemma, this result leads to a second-order parabolic PDE for the price of any path-independent financial derivative of a European style. By either solving this equation or using the risk-neutral valuation, we obtain, in particular, the celebrated Black-Scholes pricing formula for the call option. Subsequently, we show how to extend their approach to pricing and hedging of foreign market derivatives. We also examine in some detail the pricing and exercising problems for contracts of American style. A large part of this course is devoted to extensions of the Black-Scholes model, specifically, the CEV model, the local volatility model calibrated to market data, and stochastic volatility model. The course concludes by an analysis of the issue of robustness of the Black-Scholes approach.

Units of study in Semester 2: At least one of these units should be taken.

MATH4512 Stochastic Analysis Prof Ben Goldys

Capturing random phenomena is a challenging problem in many disciplines from biology, chemistry and physics through engineering to economics and finance. Hence it is of crucial importance to equip an applied mathematician with tools used to analyse and quantify random phenomena. Modern theory of financial markets relies on advanced mathematical and statistical methods that are used to model, forecast and manage risk in complex financial transactions. After the publication of the ground-breaking paper of Black and Scholes (1973) on arbitrage pricing of European options, it became clear that Stochastic Analysis and the theory of martingales are indispensable tools for the theory of arbitrage-free financial markets, derivation of prices of standard options and other derivative securities, and hedging of financial risks. In this unit, you will get familiar with the basic concepts and techniques of Stochastic Analysis, such as: the Brownian motion, continuous (local) martingales, the Itô stochastic integral, the Itô formula, stochastic differential equations, stochastic exponential, equivalent change of a probability measure and the Girsanov theorem, integral representation of martingales with respect to a Brownian filtration, relations to second order partial differential equations, and the Feynman-Kac formula.

MATH4513 Topics in Financial Mathematics Prof Marek Rutkowski

The fixed-income market is the sector of the global financial market on which various interest rate- sensitive instruments, such as: bonds, swaps, swaptions and caps, are traded. Pricing and hedging of interest rate derivatives is an important and complex issue, which creates a demand for sophisticated stochastic models capable of dealing with all kinds of interest rate risks. Theoretical term structure models are often formulated in terms of interest rates that are different from the conventional market rates. The first goal of this unit is to study various kinds of interest rates, such as: the short-term rate, the instantaneous forward rates, LIBORs and swap rates. We build stochastic models for theoretical and quoted interest rates and we examine mathematical techniques used to value and hedge fixed income securities and related derivatives. The second part of this unit is devoted to mathematical tools used for the valuation and hedging of defaultable claims, such as: vulnerable options and corporate bonds, and various classes of credit derivatives, such as: credit default swaps (CDSs) and collateralised debt obligations (CDOs). We first examine the main developments within the so-called structural approach to modelling and valuation of credit risk. In particular, we analyse Merton's model of corporate debt and the first-passage-time approach due to Black and Cox. Next, we study the so-called hazard rate approach to modelling of credit events and dependent default times, and credit ratings. Subsequently, we examine hedging strategies for multi-name credit derivatives, such as first-to-default swaps, under the assumption that some credit-risky securities (such as, e.g., corporate bonds or single-name credit default swaps).

Final Honours Mark

The final honours mark SCIE4999 for each student is based on the following marking scheme:

- **50% for the project unit FMAT4103–4106,**
- **50% for 4 units of study (12.5% for each).**

The marking scale for honours is significantly different from the undergraduate marking scale at the University of Sydney. The project will be marked with this scale in mind.

GRADE OF HONOURS	FACULTY SCALE
First Class, with Medal	95–100
First Class (possibly with Medal)	90–94
First Class	80–89
Second Class, First Division	75–79
Second Class, Second Division	70–74
Third Class	65–69
Fail	00–64

All assessable student work (such as assignments and projects) should be completed and submitted by the advertised date. If this is not possible, approval for an extension should be sought *in advance* from the lecturer concerned or, in the case of projects, from the honours coordinator. Unless there are compelling circumstances, and approval for an extension has been obtained in advance, late submissions will attract penalties (taking into account any applications for special consideration).

Project Assessments

Each student is required to complete the project unit FMAT4103–4106, which is composed of a written thesis and a seminar presentation. Students should enrol in two project units in each semester: FMAT4103–4104 in Semester 1 and FMAT4105–4106 in Semester 2.

The final mark for the project will be awarded according to the following marking scheme:

- **90% for a written thesis,**
- **10% for a seminar presentation on the project.**

The thesis will be marked by 3 different markers and each marking will therefore constitute 30% of the final project mark. The seminar is an opportunity for each student to present the material of her/his research project to members of Applied Mathematics Research Group. The seminar talk will usually be of 25 minutes duration, with an additional 5 minutes set aside for questions. The presenter of the best talk will be awarded the Chris Cannon Prize.

Project Guidelines

- A significant part of the honours program is the completion of an honours project by each student. Each student must choose a project supervisor who is willing to supervise the student's research on the topic of her/his project. The supervisor must be a member of the staff of the School of Mathematics and Statistics. Project topics and supervisors should be finalised by the beginning of Semester 1.
- The student should consult the supervisor on a regular basis, preferably at least once a week. This is the student's responsibility.
- A realistic schedule for work on the project should be drawn up at an early stage, and adhered to as closely as possible. If it proves necessary to modify the original plans, a revised schedule should be drawn up after discussion with the supervisor.
- At the end of Semester 1, a one page report has to be submitted to the Honours Coordinator. This report includes a half page description about the students aim/scope of the project and a half page description about what the student has achieved in Semester 1 and what the student wants to achieve in Semester 2. This report has to be approved by the supervisor before submission.
- The project should be based on some four to six original primary source articles, which themselves represent a substantial contribution to the topic. Secondary sources, such as books, review papers, etc., should also be consulted and cited.
- The thesis should be both a discursive and a critical account of the selected topic. It should be written at a level that an expert Applied Mathematician can be expected to understand. The work must contain substantial mathematical content.
- The length of the written thesis should be between 40 to 60 typed A4 pages. Only in exceptional circumstances, and after consultation with the supervisor, should the project exceed 60 pages. This number includes all figures, contents pages, tables, appendices, etc. Computer programs essential to the work should be included (with adequate commentary) as additional material.
- Students should be careful to provide full and correct referencing to all material used in the preparation of projects. Be explicit in stating what is your contribution and what is someone else's contribution. Avoid quoting verbatim unless you wish to reinforce an important point.
- Three examiners will be appointed to assess each written thesis. Although marking schemes may differ, the assessment of the thesis will be based on:
  - (i) selection and synthesis of source material;
  - (ii) evidence of understanding;
  - (iii) evidence of critical ability;
  - (iv) clarity, style and presentation;
  - (v) mathematical and/or modelling expertise.
- Students are advised to read the pamphlet entitled "*Guide to Essay Writing for Science Students*" available from the Science Faculty Office. Students are recommended to use LaTeX in typesetting their projects. This is available from the computers at the School.

Selected Topics of Honours Projects

You will find below a list of possible project topics for honours students in Financial Mathematics and Statistics in 2022. Prospective honours students who are interested in any of these research topics are encouraged to discuss them with the named supervisors as early as possible.

Let us stress the list is not exhaustive and thus you may wish to suggest your own topic for project or discuss any other topic with a potential supervisor. However, each student must find a member of staff who will agree to supervise the project before applying for admission to honours in Financial Mathematics and Statistics.

If you are interested in project on any topic from Financial Mathematics, feel free to contact Dr Anna Aksamit, Dr Jie Yen Fan, Prof Ben Goldys, Prof Marek Rutkowski and Dr Zhou Zhou. Their email addresses (name.surname@sydney.edu.au) and research interests can be found here.

Asymmetric information in modelling of financial markets — Dr Anna Aksamit

Differing levels of information available to agents are an essential aspect of decision-making problems. It is natural to conjecture that an agent who possesses more information in a financial market typically gains advantage over less-informed agents. In this project, you will investigate mathematical framework used to formalise the idea of different levels of information available to agents in a stochastic model of a financial market and quantify the role of information in various decision-making problems. The essential tool to address these problems is the enlargement of filtration theory, which is an actively researched area of stochastic analysis. You will learn the recent results from that theory and you will apply them to solve utility maximisation problems incorporating two distinct information flows. The related literature includes the recent monograph by Aksamit and Jeanblanc (2017) and the paper by Chau et al. (2018).

Optimisation problems for stochastic stopping games — Dr Anna Aksamit

Game theory, which focuses on solutions to optimisation problems with interactions between agents, has undoubtedly plenty of applications in various fields, in particular, economics and finance. In this project, you will study a particular class of two-player zero-sum stopping games, which are also known as Dynkin games, where the players choose their respective optimal stopping rules. The main theoretical goal of the project/essay is to investigate the most essential notions with optimal stopping rules, such as the value of the game (notice that even the existence of the value of the game is by no means ensured a priori) and the classical concept of the Nash equilibrium. In the next step, it is also possible to analyse applications to arbitrage-free pricing of some classes of game options. You may pursue to investigate several variations of the game, including asymmetric information or the case of agents with heterogeneous beliefs. The related literature includes the recent papers by Grün (2013), Ekström et al. (2017), Esmaeeli et al. (2018) and Gensbittel and Rainer (2018).

Exploration and exploitation in reinforcement learning — Dr Anna Aksamit and Dr Zhou Zhou

The project/essay focuses on stochastic control problem where an agent is choosing the best action to maximize their reward in random environment. Instead of classical setting where parameters of a random model are fully known, you will study the reinforcement learning setting where the underlying model is unknown and hence dynamic learning is needed, the agent employs exploration to interact with and learn the unknown environment through trial and error. There is a trade-off between exploration of a black box environment and exploitation of current knowledge. The agent must balance between greedily exploiting what has been learned to choose actions, and continuously exploring the environment to acquire more information to potentially achieve long-term benefits. This project will be based on the papers *Exploration versus exploitation in reinforcement learning: A stochastic control approach* and *Continuous-time mean-variance portfolio selection: A reinforcement learning framework* by Wang et al. (2019).

Population processes — Dr Jie Yen Fan

Population dynamics is the study of how and why populations change in size and structure over time. Important factors in population dynamics include rates of reproduction, death and migration. Population dynamics plays an important role in many fields, from cancer research to social science and evolutionary biology. With over two centuries of history in modelling of population dynamics, many different models have been developed. In this project, you will study one or more types of population models. There are several possible directions of this area, depending on your preference and interest. Basic knowledge of stochastic processes is assumed.

Stochastic PDEs in credit risk modelling — Prof Ben Goldys

Pricing of credit derivatives is one of the most challenging problems of Mathematical Finance. A standard tool to build models in this area is the theory of copula functions. A major deficiency of this method is that it is difficult to incorporate dynamics in the model. This problem has been recently addressed by using the mean-field method well known in Physics in the paper by N. Bush et al.: *Stochastic evolution equations in portfolio credit modelling,* SIAM Journal on Financial Mathematics 2 (2011), 627–664. Using this approach we can derive a stochastic partial differential equation that describes time evolution of the fraction $f(t, x)$ of agents that did not default before a given time and are in a distance x to the default. The aim of this project is to study this approach and develop more realistic models.

SDEs with memory and applications to stochastic volatility models — Prof Ben Goldys

A standard assumption of the Black-Scholes model for pricing of options is that the volatility of the asset, if random, is driven by Wiener process. However, recent careful analysis of market data show that the volatility is a much rougher process than the Wiener process (see, for example, J. Gatheral, T. Jaisson, M. Rosenbaum: *Volatility is rough*, Quantitative Finance 18 (2018), 933-949. In this project we will investigate a simple model of stock prices in the case when volatility is modelled as a solution to a stochastic differential equation (SDE) with memory. The main question will be how to derive (or approximate) prices and hedges for financial derivatives.

Designing optimal contracts

Prof Ben Goldys

Contract theory is part of economics focused on designing optimal contracts between principals and agents. Only recently, mathematical methods have become available in this area. The breakthrough has been made by Y. Sannikov in his seminal paper *A continuous-time version of the principal-agent problem*, Review of Economic Studies 75 (2008), 957–984. The principal-agent problem occurs when one person or entity (the *agent*) is able to make decisions and/or take actions on behalf of another person (the *principal*). The dilemma exists in circumstances where agents are motivated to act in their own best interests, which are contrary to those of their principals. The proposed solution to the conundrum is to ensure the provision of appropriate incentives so agents act in the way principals wish. We will study Sannikov's paper and will try to extend its conclusions to more realistic situations. Notice that familiarity with stochastic analysis is necessary for this project.

Representations of American options via European claims

Prof Marek Rutkowski

A challenging mathematical problem of great importance for the Finance industry was formulated by Jourdain and Martini (2001): they were interested in pricing of path-independent European claims in the Black-Scholes model and they discovered that, for a large class of European payoffs, in the region where the European price increases with the time to maturity, this price is equal to the American price of another claim. They also furnished particular examples where the corresponding American claims can be computed explicitly. However, the full characterisation of American claims obtained in this way was, and still remains, an open question, although a first step towards verifying the representability of certain American claims was recently done by Lenga (2017). As shown by Jourdain and Martini (2002), some preliminary theoretical results concerning the problem of representation of an American option through a European claim can also be used to obtain new approximation schemes for prices of American options. Recall that in the Black-Scholes model, no closed-form expression is available even for the price of the standard American put and thus numerous numerical approximation methods have been specifically designed for this problem. This project/essay combines an intriguing theoretical problem with numerical aspects of arbitrage-free pricing of American options and thus it is particularly suitable for a student with a working knowledge of computational methods.

Analytical valuation under funding costs and credit risk

Prof Marek Rutkowski

The credit valuation adjustment (CVA) and the funding valuation adjustment (FVA) are the price adjustments due to the default risk and the cost of funding the trade. Trading desks back the deal with a client by hedging it with other dealers in the market, and this may involve maintaining a number of hedging accounts in the underlying assets, in cash, or in other correlated assets when proxy hedging. The funds needed for these operations are raised from the internal treasury of the dealer and, ultimately, they come from external funders. Interest charges on all borrowing and lending activities need to be covered and this affects the contract's valuation. The causes of these adjustments are accounted for at the level of the contract payoffs and the resulting all-inclusive price is written as a solution to an advanced mathematical problems, such as semilinear PDEs or BSDEs (see Bielecki et al. (2018)). The challenging theoretical goal is to identify classes of contracts and market models where this all-inclusive price of an uncollateralized contract can be computed analytically.

Pricing of superannuation guaranteed benefits

Prof Marek Rutkowski

Superannuation guaranteed benefits resemble in some respects American and game options, but their structure is even more convoluted since the holder can make decisions about withdrawals. The goals of the project differ from the classical portfolio optimisation since superannuation guarantee products have several non-standard features: a specific design of the income stream and gains from market upside, in some instances, optionality of withdrawals, a possibility of termination by the holder and, finally, death benefits for designated beneficiaries. In contrast, classical portfolio optimisation problems focus on either achieving the desired level of expected rate of return on a dynamic portfolio of assets or maximisation of the expected utility from the terminal wealth. The superannuation guarantee product need to be valued upfront and then hedged by the insurance provider using highly sophisticated trading strategies to avoid losses in the event of market downturn whereas the standard portfolio optimisation assumes that the initial wealth is given. For the existing literature, see Forsyth and Vetzal (2014), Huang, Zeng and Kwok (2017) and Luo and Shevchenko (2017).

Utility maximisation under model uncertainty

Dr Zhou Zhou

One of the most important and popular problems in financial mathematics is the utility maximisation, where an investor needs to find the best trading strategies for stocks in order to maximise the expected payoff. In the classical setup for utility maximisation, it is assumed that the investor fully knows the underlying market model including the dynamics of the stock prices, which, in reality, can never be true. In this project, you will consider the utility maximisation problem under model uncertainty, where the investor takes all possible models into consideration when seeking optimal trading strategies. The aim of this project is to establish the duality result for the value function and to characterise the optimal solution. The project will be based on the papers *The asymptotic elasticity of utility functions and optimal investment in incomplete markets* by Kramkov and Schachermayer, and *Duality theory for robust utility maximization* by Bartl et al..

Dynamics for the set value of a non-zero-sum game

Dr Zhou Zhou

In reality individuals, companies, and/or governments (players) interact with each other, and game theory is a powerful tool to study such interaction and seek optimal strategies. A non-zero-sum game refers to a game where one player's gain does not necessarily result in another player's loss (thus win-win could be a possible outcome). A non-zero-sum game may have multiple Nash equilibria, and different equilibria may lead to different values and thus the set value. This set value admits several nice properties such as certain continuity and stability. In this project, you will investigate a critical property, dynamic programming principle, for the set value of a non-zero-sum game. The aim of this project is to establish the dynamic programming principle and provide an efficient computational method for the set value. This project will be based on the paper *Dynamic set values for nonzero sum games with multiple equilibriums* by Feinstein et al..

Mean-field ranking games

Dr Zhou Zhou

Companies or individuals interact with each other, and it is of particular interest how they choose strategies in order to maximise their own profit/payoffs. Such problems can be modelled as games. Traditional methods of game theory assume that each player keeps an eye on every other player. As a result, roughly speaking, the dynamics of an n-player game can be represented by a system of (at least) n coupled differential equations. Unfortunately, when n is relatively large, the system of n equations are usually computationally intractable. On the other hand, in practice when there are a lot of players, each player may not keep track of every other player's behaviour. More likely, she may only pay attention to the aggregate behaviour of the cohort of other players. The above intractability and practical situation motivates us to seek an effective device to study games with a large number of players, where each player's payoff depends on relevant features of the aggregate behaviour of other players. The tool of mean-field games serves this purpose.

In this project, you will study a mean-field ranking game, where players' rewards are determined by the ranking of their performance and are subject to cost of effort. You will calculate the Nash equilibria, and design the reward functions in order to achieve desired equilibria. This project is based on the papers *Large tournament games* by Bayraktar et al., and *Terminal ranking games* by Bayraktar and Zhang.

Machine learning for solving PDEs — Dr Zhou Zhou

Most PDEs cannot be solved explicitly. In this case what we can do is to find the solutions numerically. It has always been a challenge to numerically solve high-dimension PDEs due to the so called "curse of dimensionality". In fact, traditional approaches such as finite difference method cannot be effectively implemented for a nonlinear PDE with dimension above 4. Very recently some new methods, including machine learning, are developed in order to overcome this difficulty. In this project, you will consider machine learning schemes for solving high-dimension nonlinear PDEs based on the deep neural network technique and BSDE theory. You will rigorously analyse the convergence for the numerical schemes. The project is based on the papers *Deep backward schemes for high-dimensional nonlinear PDEs* by Huré et al., and *Neural networks-based backward scheme for fully nonlinear PDEs* by Pham et al.

Portfolio optimisation with liquid American options — Dr Zhou Zhou

As a classical problem in mathematical finance, portfolio optimisation has been studied extensively in various setups. American-style options, with huge trading volumes, are perhaps the most popular financial derivatives in the financial market. However, so far there have been very limited studies on portfolio optimisation involving liquidly traded American options. Among the few works, Bayraktar and Zhou (2016) analyses a general utility maximisation problem with the short-selling constraint on liquid American options. They establish a duality result for the value function associated with the utility maximisation problem. In this project/essay, you will consider the portfolio optimisation in which stocks are traded dynamically and American options are traded statically. Different from Bayraktar and Zhou (2016), here we assume that American options can be sold. Due to the nature of American options, investors who short the options will face the uncertainty of the exercise times. How to deal with this uncertainty and what are the corresponding trading strategies will be the core questions for this project/essay. In addition to probability, stochastic processes and control, the project will also involve game theory. The project is particularly suitable for students who are interested in game theory and/or its applications. For some relevant works, see Bayraktar and Zhou (2014, 2017).

Prizes and Awards

The following prizes may be awarded to Applied Mathematics Honours students of sufficient merit. Students do not need to apply for these prizes, which are awarded automatically. A complete list of the prizes and scholarships offered by the School of Mathematics and Statistics can be found here.

Joye Prize in Mathematics — Value: **$5300, with medal and shield**

Awarded to the most outstanding student completing Honours in the School of Mathematics and Statistics.

University Medal

Awarded to honours students who perform outstandingly. The award is subject to Faculty rules, which require a Faculty mark of 90 or more in Applied Mathematics 4 and a Third Year WAM of 80 or higher. A medal is always awarded when the Faculty mark is 95 or higher. More than one medal may be awarded in any year.

K. E. Bullen Memorial Prize — Value: **$1000**

Awarded annually on the recommendation of the Head of the School of Mathematics and Statistics in consultation with the professors of Applied Mathematics to the most proficient student in Applied Mathematics 4, provided that the student's work is of sufficient merit.

Barker Prize — Value: **$550**

Awarded at the Fourth (Honours) Year examination for proficiency in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

M. J. and M. Ashby Prize — Value: **$360**

Offered annually for the best project, submitted by a student in the Faculty of Science, that forms part of the requirements of Pure Mathematics 4, Applied Mathematics 4 or Mathematical Statistics 4.

Norbert Quirk Prize No IV — Value: **$250**

Awarded annually for the best project on a given mathematical subject by a student enrolled in a Fourth Year course in mathematics (Pure Mathematics, Applied Mathematics or Mathematical Statistics) provided that the essay is of sufficient merit.

Australian Federation of Graduate Women Prize in Mathematics. — Value: **$175**

Awarded annually, on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate for the degree of BA or BSc who graduates with first class Honours in Applied Mathematics, Pure Mathematics or Mathematical Statistics.

Chris Cannon Prize — Value: **$100**

For the best adjudged project seminar presentation of an Applied Mathematics 4 student.

Rights and Responsibilities

Honours students will have access to the following:

- Office space and a desk in the Carslaw building.
- A computer account with access to e-mail and the World Wide Web, as well as TEX and laser printing facilities for the preparation of projects and essays.
- A photocopying account paid by the School for assembling project source material.
- After-hours access to the Carslaw building.
- A pigeon-hole in room 728.
- Participation in the School's social events.
- Class representative at School meetings.

Honours students have the following obligations:

- Have regular meetings with project supervisors and meet all deadlines.
- Regular attendance at the weekly seminars *Stochastics and Finance*.
- Contribute towards the academic life in Financial Mathematics at the School of Mathematics and Statistics.
- Utilise all School resources in an ethical manner.

Life After Fourth Year

Postgraduate Studies

Many students completing the Honours programme have in the past gone on to pursue postgraduate studies at the University of Sydney, at other Australian universities, and at overseas universities. Please see the School's Coordinator of Postgraduate Studies if interested in enrolling for a MPhil or PhD at the School of Mathematics and Statistics. Students who do well in Applied Mathematics Honours may be eligible for postgraduate scholarships, which provide financial support during subsequent study for higher degrees at Australian universities. The honours coordinator is available to discuss options and provide advice to students interested in pursuing studies at other universities.

Careers

Students seeking assistance with post-grad opportunities and job applications should feel free to ask lecturers most familiar with their work for advice and written references. The Director of the Applied Mathematics Teaching Programme, the Course Coordinator and the unit lecturers may also provide advice and personal references for interested students.

Important Dates

Semester 1: 21 February 2022 to 29 May 2022

- **Seminar presentation: Thursday and Friday in week 8**
- **Project submission: Monday, 23 May 2022 (week 13)**
  For students completing in Semester 1, 2022. An electronic file (pdf format) must be uploaded on Canvas before the deadline.
- **Examinations: 6–18 June 2022**

Semester 2: 1 August 2022 to 6 November 2022

- **Seminar presentation: Thursday and Friday in week 8**
- **Project submission: Monday, 31 October 2022 (week 13)**
  For students completing in Semester 2, 2022. An electronic file (pdf format) must be uploaded on Canvas before the deadline.
- **Examinations: 14–26 November 2022**