AUSTRALIA-CHINA JOINT CONFERENCE ON PARTIAL DIFFERENTIAL EQUATIONS

11–15 DECEMBER 2023, SYDNEY, AUSTRALIA

Abstracts of Invited Talks

Ben Andrews (Australian National University)
Multipoint methods for heat flows in geometry
Abstract: I will discuss some sharp estimates for regularity of heat flows which apply in a variety of geometric settings, including sharp gradient and Hessian estimates, Harnack estimates and estimates on moduli of continuity.

Serena Dipierro (University of Western Australia)
The Allee effect: cooperation and competition among biological populations
Abstract: We present some recent results describing a logistic model for a biological population subject to classical and Lévy dispersals in an ecological niche. The population is assumed to possess a cooperative behaviour and we analyse the chance of survivals in dependence of the structural parameters.

Zhewen Feng (University of Queensland)
Local well-posedness of the Beris-Edwards system with general Landau-de Gennes energy in dimension three
Abstract: The Beris-Edwards system is a strong coupling between Navier-Stokes equations and the gradient flow of Landau-de Gennes energy. In this presentation, we will briefly review recent results on Q-tensor theory and discuss the Beris-Edwards system for uniaxial Q-tensors, with a general Landau-de Gennes energy density that depends on four non-zero elastic constants. In addition, we will be presenting our recent collaborative work with Min-Chun Hong and Yu Mei. Our study establishes the existence of solutions for the uniaxial Beris-Edwards system until a maximal time using scaling analysis. Furthermore, we shall demonstrate the smooth convergence of solutions for a penalised Beris-Edwards system to the solution of the uniaxial Beris-Edwards system up to the maximal existence time.

Ben Goldys (University of Sydney)
Linear parabolic PDEs with very rough boundary conditions
Abstract: We will consider an approach to linear parabolic equations with the Dirichlet boundary conditions given by derivatives of Brownian motion depending on space and time. We will first prove the existence and uniqueness of solutions in appropriately defined weighted L^p spaces. Then we will present some properties of solutions, including smoothing properties and large time behaviour. Finally, we will derive gradient and Hessian estimates for the associated transition semigroup.

Yong Huang (Hunan University)

Riesz potential, chord measure in integral geometry and their Minkowski problems

Abstract: In this talk, we discuss the chord Minkowski problem recently posed by Lutwak-Xi-Yang-Zhang, which characterizes integral geometric invariants of convex bodies. They also obtained the weak solution in the sense of Alexandrov. From point of partial differential equation view and based on a new technique for obtaining smooth estimation of a Riesz potential on the boundary, we make use of an anisotropic nonlocal Gauss curvature flow to develop the regularity theory of a new nonlocal Monge-Ampère equations. When goes to infinity, we find that it recovers a smooth solution of the chord Minkowski problem. That is a joint work with Jinrong Hu, Jian Lu and Sinan Wang.

Brian Krummel (University of Melbourne)

Analysis of singularities of area minimizing currents

Abstract: In his monumental work in the early 1980s, Almgren showed that the singular set of an n-dimensional locally area minimizing submanifold T has Hausdorff dimension at most $n-2$. The main difficulty is that higher codimension area minimizers can admit branch point singularities, i.e. singular points at which one tangent cone is a plane of multiplicity two or greater. Almgren's lengthy proof showed first that the set of non-branch-point singularities has Hausdorff dimension at most $n-2$ using an elementary argument based on tangent cone type, and developed a powerful array of ideas to obtain the same dimension bound for the branch separately. In this strategy, the exceeding complexity of the argument stems largely from the lack of an estimate giving decay of T towards a unique tangent plane at a branch point.

We will discuss a new approach to this problem (joint work with Neshan Wickramasekera). In this approach, the set of singularities (of a fixed integer density q) is decomposed not as branch points and non-branch-points, but as a set $\mathcal{B}$ of branch points where T decays towards a (unique) plane faster than a fixed rate, and the complementary set $\mathcal{S}$. Using a new intrinsic frequency function for T relative to a plane and a blow-up method of L. Simon and Wickramasekera, we show that T has a unique non-planar tangent cone at $\mathcal{H}^{n-2}$-a.e. point of $\mathcal{S}$ and T is asymptotic to a unique homogeneous harmonic multi-valued function at $\mathcal{H}^{n-2}$-a.e. point of $\mathcal{B}$. It follows that the singular set of T is in fact countably $(n-2)$-rectifiable.

Haigang Li (Beijing Normal University)

Babuska problem in composite materials and suspension problem in Stokes flow

Abstract: In high-contrast elastic composite media, when inclusions are spaced closely, the stress always concentrates in between inclusions and causes damage initiation. For Babuska problem in linear elasticity, we obtain the blow-up asymptotic expressions of the gradients and second derivative estimates of the solutions to the Lamé system with partially infinite coefficients in the narrow region when the distance between inclusions tends to zero. This builds a bridge between PDE analysis and numerical computation. Several numerical experiments in two and three dimensions are demonstrated. By taking advantage of the iteration method for the energy developed by us, the stress concentration problem between two convex adjacent rigid particles suspending in Stokes flow is solved in dimensions two and three.

Haizhong Li (Tsinghua University)
Hyperbolic p-sum and horospherical p-Brunn-Minkowski theory in hyperbolic space
Abstract: The classical Brunn-Minkowski theory studies the geometry of convex bodies in Euclidean space by use of the Minkowski sum. It originated from H. Brunn's thesis in 1887 and H. Minkowski's paper in 1903. Since there is no universally acknowledged definition of the sum of two sets in hyperbolic space, there has been no Brunn-Minkowski theory in hyperbolic space since 1903. In this talk, for any $p > 0$ we introduce a sum of two sets in hyperbolic space, and we call it the hyperbolic p-sum. Then we develop a Brunn-Minkowski theory in hyperbolic space by use of our hyperbolic p-sum, and we call it the horospherical p-Brunn-Minkowski theory. This is joint work with Botong Xu.

Junbin Li (Sun Yat-sen University)
On the instability of naked singularities in general relativity
Abstract: The weak cosmic censorship, one of the central open problems in general relativity, states that naked singularities cannot appear in gravitational collapse generically. In this talk, I will report some research progress on the instability of naked singularities.

Ji Li (Macquarie University)
Flag Hardy space theory—a complete answer to a question by E.M. Stein
Abstract: The theory of multiparameter flag singular integral originates from the study of $\bar{\partial}$-problem on the Heisenberg group by D. Phong and E.M. Stein. In 1999, E. M. Stein raised the question "What is the Hardy space theory in the flag setting?" in the conference at Washington University in Saint Louis. In our recent work, we established a complete flag Hardy space theory on the Heisenberg group. It provided a unified approach for proving the L^p boundedness of different types of singular integrals, and led to the endpoint $L\log L \to L^{1,\infty}$ estimates. The real analysis approach in our work also provides a feasible tool for addressing the $\bar{\partial}$-Neumann problem on decoupled boundaries in $\mathbb{C}^{n+1}$.

Yuan Lou (Shanghai Jiao Tong University)
Basic reproduction number and principal eigenvalue
Abstract: Basic reproduction number is a dimensionless constant which is used in epidemiology to determine if an emerging infectious disease can spread. Principal eigenvalue, a key concept in spectral theory, is used to reflect certain properties of matrices or differential operators. In this talk we will discuss some recent progress on principal eigenvalues for second-order linear elliptic and time-period parabolic operators, with applications to basic reproduction number for some PDE models for infectious disease.

Xi-Nan Ma (University of Science and Technology of China)
Liouville theorem for elliptic equations involving the sum of the function and its gradient in $\mathbb{R}^n$
Abstract: We prove Liouville theorem for the equation $\Delta v + Nv^p + M|\nabla v|^q = 0$ in $\mathbb{R}^n$, with M, $N > 0$, in the critical and subcritical cases. The proof is based on an integral identity and Young inequality. This is the joint work with Wangzhe Wu and Qiqi Zhang.

Qintang Su (Chinese Academy of Sciences)

The global well-posedness of 2d water waves with a pair of point vortices

Abstract: We prove the global well-posedness of water waves with a pair of point vortices move with almost constant velocity in the direction away from the free boundary. We obtain uniform in time control of the energy functional and show that the asymptotic behavior of the point vortices corresponds to two straight lines parallel to the y-axis.

Tom ter Elst (University of Auckland)

The Dirichlet problem for elliptic equations without the maximum principle

Abstract: The maximum principle plays an important role for the solution of the Dirichlet problem. Now consider the Dirichlet problem with respect to the elliptic operator

$$-\sum_{k,l=1}^{d}\partial_k\, a_{kl}\,\partial_l-\sum_{k=1}^{d}\partial_k\, b_k+\sum_{k=1}^{d}c_k\,\partial_k+c_0$$

on a bounded open set $\Omega\subset\mathbb{R}^d$, where $a_{kl},c_k\in L_\infty(\Omega,\mathbb{R})$ and $b_k,c_0\in L_\infty(\Omega,\mathbb{C})$. Suppose that the associated operator on $L_2(\Omega)$ with Dirichlet boundary conditions is invertible. Note that in general this operator does not satisfy the maximum principle. We define and investigate a solution of the Dirichlet problem with data in $C(\partial\Omega)$. We show that it coincides with the Perron solution, in case the maximum principle would be available. In the general setting we characterise this solution in different ways: by approximating the domain by smooth domains from the interior, by variational properties, by the pointwise boundary behaviour at regular boundary points. We also investigate for which boundary data the solution has finite energy. We show that the solution is obtained as an H_0^1-perturbation of a continuous function on $\overline{\Omega}$. This is new even for the Laplacian.

Enrico Valdinoci (University of Western Australia)

Nonlocal capillarity theory

Abstract: We describe some recent results motivated by a nonlocal theory of capillarity, as related to the formation of droplets due to long-range interaction potentials. We will discuss the notion of contact angle in this setting, considering a nonlocal version of the classical Young's Law, together with some regularity and asymptotic properties.

Valentina Wheeler (University of Wollongong)

The vanishing fundamental gap in hyperbolic space

Abstract: The talk will present some background and history about the fundamental Gap conjecture and major results around it from my non expert perspective and focus at the end on presenting the counterexample of the gap existence for convex domains in hyperbolic space.

Zhouping Xin (The Chinese University of Hong Kong)
On the Prandtl's Boundary Layer Theory for Steady Sink-Type Flows
Abstract: In this talk, I will present some results on the large Reynolds number limits and asymptotic behaviors of solutions to the steady incompressible Navier-Stokes equations in two-dimensional infinitely long convergent nozzles. The main results show that the Prandtl's laminar boundary layer theory can be rigorously established and the sink-type Euler flow superposed with a self-similar Prandtl's boundary layer flow is shown to be uniformly structurally stable as long as the viscous flow has a given negative mass flux and the boundaries of the nozzle satisfy a curvature decreasing condition. Furthermore, the asymptotic behaviors of the solutions at both the vertex and infinity can be determined uniquely which plays a key role in the stability analysis. Some of key ideas in the theory will be discussed. This talk is based on a joint work with Dr. Chen Gao.

Shusen Yan (Central China Normal University)
On the critical points for the Robin functions
Abstract: The study of blow-up solutions for many nonlinear elliptic problems will lead to the investigation of the non-degenerate critical points of the Robin functions. In this talk, I will present some results on this aspect, with emphasis on the effects of the small holes in the domain on the existence and non-degeneracy of the critical points.

Yingying Zhang (Tsinghua University)
Complex deformation of Fano Kähler-Einstein manifolds
Abstract: The problem of studying the deformation of complex structures goes back to Riemann and was later developed in general by Kodaira, Spencer, Kuranishi, etc. It plays an important role in understanding the local geometry of the moduli space of a complex manifold. In this talk, we will discuss the complex deformation theory of compact Fano Kähler-Einstein manifolds. This amounts to study the Monge-Ampère equation coupling with a Maurer-Cartan equation. We will describe sufficient and necessary geometric conditions leading to the solvability of this deformation problem. We will also apply the established deformation theory to study the Weil-Petersson geometry on the moduli space of a Fano Kähler-Einstein manifold. In particular, we will show that the Weil-Petersson metric can be approximated by the Ricci curvature of the L^2 metric on the direct image sheaf associated to the relative pluri-anticanonical line bundle over the moduli space.

Zhifei Zhang (Peking University)
Nonlinear inviscid damping for the 2-D inhomogeneous Euler equations
Abstract: We prove the asymptotic stability of shear flows close to the Couette flow for the 2-D inhomogeneous Euler equations. More precisely, if the initial velocity is close to the Couette flow and the initial density is close to a positive constant in the Gevrey class 2, then the 2-D inhomogeneous Euler equations are globally well-posed and the velocity converges strongly to a shear flow close to the Couette flow, and the vorticity will be driven to small scales by a linear evolution and weakly converges as $t \to \infty$. This is the first global well-posedness result for the 2-D inhomogeneous Euler equations.

Chunqin Zhou (Shanghai Jiao Tong University)

The recent developments of the super-Liouville equations

Abstract: In this talk, I will introduce the super-Liouville equations, arising from Liouville field theory in supergravity. Recall the classical Liouville field theory describes the matter-induced gravity in dimension two, the super-Liouville field theory is a supersymmetric generalization of the classical one, by taking the spinorial super-partner into account, so that the bosonic and fermionic fields couple under the supersymmetry principle. For super-Liouville equations on closed Riemannian surface, they are conformal invariant and have a variational structure with a strongly indefinite functional. We will talk about the blow-up analysis of this equations. And further some existence results will be introduced.

Xi-Ping Zhu (Sun Yat-sen University)

Weyl's lemma on RCD metric measure spaces

Abstract: The classical Weyl's lemma states that any very weakly harmonic function on the Euclidean spaces must be smooth. In this talk, we extend the Weyl's lemma to RCD metric measure spaces. As its applications, we show the regularity of very weak solutions for Poisson equations and a Liouville-type result for L1 very weakly harmonic functions on RCD spaces. This is a joint work with Yu Peng and Hui-Chun Zhang.