

THE UNIVERSITY OF SYDNEY
MATH2969 GRAPH THEORY

Semester 1	Assignment	2008
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This Assignment is due on **Monday 26 May 2008**. Hand in at our lecture. Staple your solutions into a folder, bearing your name and SID. Include the initial letter of your surname in a LARGE letter on the front. I hope you enjoy this assignment. Feel free to come and discuss any or all of the questions at a consultation session.

1. Let $S = \{1, 2, 3, 4, 5\}$. Let G be the simple graph whose vertices are the 2-element subsets of S and whose edges are the pairs of disjoint 2-element subsets. For example, $\{1, 2\}$ is a vertex of G and there is an edge with endvertices $\{1, 2\}$ and $\{3, 4\}$.
 - (i) Find the order and size of G .
 - (ii) Show that G contains two distinct 5-cycles.
 - (iii) Let u and v be any non-adjacent vertices in G . Show that u and v have exactly one common neighbour.
 - (iv) The *girth* of a graph with a cycle is the length of the shortest cycle. A graph with no cycle has infinite length. So, for example, C_3 has girth 3 and a tree has infinite length.
Show that G has girth 5.
2.
 - (i) From the definition of isomorphism, prove that graphs G and H are isomorphic if and only if \overline{G} and \overline{H} are isomorphic.
 - (ii) Prove that there are exactly two (unlabelled) simple 4-regular graphs of order 7.
3. Let G be the graph with vertices labelled 1, 2, 3, 4, 5 and edges labelled e_1, \dots, e_7 corresponding to $[1, 2], [1, 5], [2, 3], [2, 4], [3, 4], [3, 5], [4, 5]$ respectively.
 - (i) Draw the graph G , with all labels appropriately attached, and write down its adjacency matrix A .
 - (ii) Use the matrix tree theorem to determine the number of spanning trees of G , and give labelled diagrams for each of these trees.
4. Prove that the complement of a simple planar graph of order at least 11 is non-planar. Construct a self-complementary simple planar graph of order 8.
5. State two alternative forms of the reduction theorem for chromatic polynomials. Use whichever seems appropriate to calculate the chromatic polynomial for the following disconnected graph.
Determine the chromatic number of the graph.

