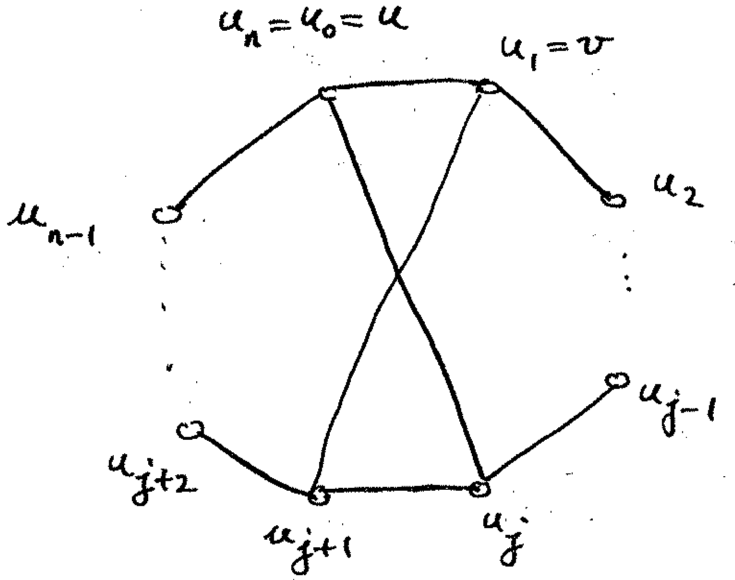
Thus, $\exists$ some $j \in U \cap V$. So $\{u, u_j\}$ & $\{v, u_{j+1}\}$ are edges.
We form the Hamiltonian cycle C' (that does not include the edge $\{u, v\}$.



$$C' = (u_0 = u, u_j, u_{j-1}, \dots, \underset{v}{\overset{}{u_1}}, u_{j+1}, u_{j+2}, \dots, u_{n-1}, u_n = u).$$

Therefore, C' is a Hamiltonian cycle of G.

Whence we have easily

<u>Ore's Theorem</u>

Let G be a simple graph of order $n \geq 3$. If, for every pair of non-adjacent vertices $u, v \in V(G)$, we have
$$\deg_G(u) + \deg_G(v) \geq n,$$
then G is Hamiltonian.

<u>Corollary</u>

If G is a simple planar graph of order $v \geqslant 3$ and size e, then

$$e \leq 3v - 6.$$

<u>Proof:</u> Add sufficiently many edges so that the resulting graph G' with v' vertices and e' edges is maximal planar.

Clearly, $v = v'$

$$e \leq e'.$$

As G' is maximal planar,

$$e' = 3v - 6$$

and so

$$e \leq 3v - 6.$$

Another great useful result follows:

<u>Corollary</u>

Every simple planar graph contains a vertex of degree ≤ 5.

We will need this corollary when we consider graph colourings.

For simple graphs, we say:
A simple graph G is of <u>class one</u>
if $\chi'(G) = \Delta(G)$.
A simple graph G is of <u>class two</u>
if $\chi'(G) = \Delta(G) + 1$.

Generally, it is not known which simple graphs belong to class one or class two.

We <u>do know</u>: $\chi'(C_{2n}) = 2$

$\chi'(C_{2n+1}) = 3$

$\chi'(W_n) = n-1, \ n \geq 4$

<u>Recall</u>: W_n is wheel graph

e.g.

W_5: