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THE UNIVERSITY OF SYDNEY
MATH2969/2069 GRAPH THEORY

Semester 1 — **Quiz 2** — 2008

SID:	**Name:**
Circle one of: MATH2969 or MATH2069	**Signature:**

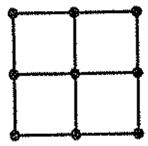
For questions **1.** to **29.**, circle the answer that you think is correct. Answer the remaining questions in the indicated parts of the question sheet.

1. A Hamiltonian cycle in a Hamiltonian graph of order 24 has

(a) 12 edges. (b) 24 edges. (c) 23 edges.

(d) none of the above

2. The graph given below is bipartite.



(a) TRUE. (b) FALSE.

3. A simple graph G with 13 vertices has 4 vertices of degree 3, 3 vertices of degree 4 and 6 vertices of degree 1.
The graph G must be a tree:

(a) TRUE. (b) FALSE.

4. A spanning tree for a simple graph of order 24 has

(a) 12 edges. (b) 6 edges. (c) 23 edges.

(d) none of the above.

5. The number of spanning trees in the complete graph K_8

(a) 48. (b) 6^8. (c) 8^6. (d) none of the above.

6. The order of a forest, F, with 17 vertices and 4 components is

(a) 17. (b) 4. (c) 16. (d) none of the above.

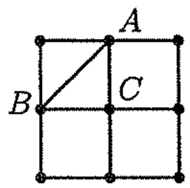
7. The size of a forest, F, with 17 vertices and 4 components is size = 13
(a) 14. (b) 4. (c) 16. (d) none of the above.

8. A forest, F, with 17 vertices and 4 components has at least one 3-clique.
(a) TRUE. (b) FALSE.

9. The number of different labelled trees of order n is
(a) n^n. (b) $(n-2)^n$. (c) n^{n-2}. (d) none of the above.

10. Consider the Prüfer sequence, $S = (1, 7, 1, 5, 2, 5)$.
Let T be the labelled tree corresponding to S.
The degree of the vertex 7 in T has degree
(a) 2. (b) 1. (c) 5.
(d) 7. (e) none of the above.

11. Consider the Prüfer sequence, $S = (1, 7, 1, 5, 2, 5)$.
Let T be the labelled tree corresponding to S.
The size of T is
(a) 2. (b) 6. (c) 5.
(d) 7. (e) none of the above.



Badly phrased question: number of closed walks = 6.

12. The number of walks of length 3 through the vertices A, B and C in the graph, G, drawn above, is
(a) 3. (b) 6. (c) 12. (d) 1. (e) none of the above.

13. The cube graph Q_5 is planar.
(a) TRUE. (b) FALSE.

14. The complete graph K_7 is non-planar.
(a) TRUE. (b) FALSE.

15. The complete bipartite graph $K_{4,3}$ is non-planar.
(a) TRUE. (b) FALSE.

16. If G is a simple connected 3-regular planar graph where every face is bounded by exactly 3 edges, then the size of G is
(a) 3. (b) 4. (c) 6.
(d) 5. (e) none of the above.

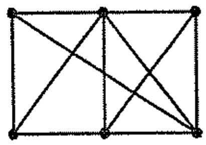
Let G be a graph.
The next three questions refer to A which is an adjacency matrix for G.

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

17. The degree sequence of G is
(a) $(2,2,3,3,4)$. (b) $(0,1,1,2,0)$. (c) $(0,0,1,1,2)$.
(d) $(2,3,4)$. (e) none of the above.

18. The size of the graph G is
(a) 2. (b) 3. (c) 7. (d) none of the above.

19. The entry at position $(4,2)$ in the matrix A^2 is
(a) 2. (b) 1. (c) 0.
(d) 4. (e) none of the above.

G :


20. Let H be the plane drawing of the planar graph G, drawn above. The graph H has
(a) 10 faces. (b) 5 faces. (c) 11 faces.
(d) 6 faces. (e) none of the above.

there are 7 faces.

The next two questions refer to the graph H drawn below:

H :


21. The size of H^*, the dual of H is
(a) 4. (b) 3. (c) 6. (d) none of the above.

22. The order of H^* is
(a) 4. (b) 3. (c) 6. (d) none of the above.

23. If G is a connected plane graph of order v, size e and with f faces, then
(a) $v - e + f = 2$. (b) $e - v + f = 2$.
(c) $v + e - f = 2$. (d) $e + v - f = 2$.
(e) none of the above.

24. The chromatic number of the cyclic graph C_{15} is

(a) 3. (b) 2. (c) 6.

(d) 15. (e) none of the above.

25. The chromatic number of the complete graph K_{15} is

(a) 3; (b) 2; (c) 6;

(d) 15. (e) none of the above.

26. The chromatic polynomial $P_G(t)$, where G is the complete graph K_3 is

(a) $t(t-1)^2$. (b) $t(t-1)(t-2)$.

(c) t^3. (d) $t(t-1)(t+1)$.

(e) none of the above.

27. The polynomial $t^4 - 4t^3 + 6t^2 - 3t$ is the chromatic polynomial for a graph G. Thus the size of G is

(a) 4. (b) 6. (c) 3.

(d) 2. (e) none of the above.

28. The polynomial $t^4 - 4t^3 + 6t^2 - 3t$ is the chromatic polynomial for a graph G. Thus the order of G is

(a) 4. (b) 6. (c) 3. (d) 2.

(e) none of the above.

29. The edge chromatic number (or chromatic index) of the complete graph K_{15} is

(a) 3. (b) 2. (c) 6. (d) 15.

(e) none of the above.

30. Dirac's Theorem is about Hamiltonian graphs.
State Dirac's Theorem.

see note p. 105

31. Copy and complete: "Every tree of order at least 2, has at least ... leaves"

see notes p. 131

32. Kuratowski's Theorem is related to planar graphs
State Kuratowski's Theorem.

see notes p. 231

33. Copy and complete: "Let G be a loopless labelled graph with adjacency matrix A and degree matrix D. Then all cofactors of ... "

see note p. 177

34. Copy and complete Brooks' Theorem. "If G be a simple connected graph with largest vertex-degree Δ, then ..."

see notes p. 269

35. Copy and complete Vizing's Theorem. "If G be a simple graph with largest vertex-degree Δ, then ..."

see notes p. 300

36. Suppose that G is a simple connected plane graph with v, e edges and f faces.
If each face of G is a 3-cycle, show that $e = 3v - 6$.

see notes p. 221

37. By using the result of the previous question, or otherwise, show that K_5 cannot be planar.

see notes p. 225.

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THE UNIVERSITY OF SYDNEY

MATH2969/2069 GRAPH THEORY

Semester 1A	Quiz 1	2008

SID:	Name:
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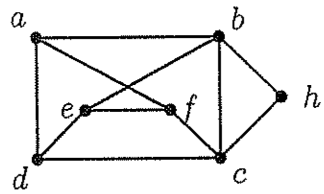
For questions **1.** to **30.**, circle the answer that you think is correct. Answer the remaining questions in the indicated parts of the question sheet.

Questions **1.** to **6.** refer to the graph G drawn below:

G :



1. The degree sequence for G is:

(a) $(2,2,3,3,3,4,4)$; (b) $(2,3,3,3,3,4,4)$; (c) $(2,3,3,3,3,3,4)$;
(d) $(2,3,3,3,4,4,4)$.

2. The size of G is: none of these. size of G = 11

(a) 10; (b) 4; (c) 7; (d) 2.

3. The order of G is:

(a) 10; (b) 4; (c) 7; (d) 2.

4. The graph G is best described as

(a) a multigraph; (b) a pseudograph; (c) a complete graph;
(d) a simple graph.

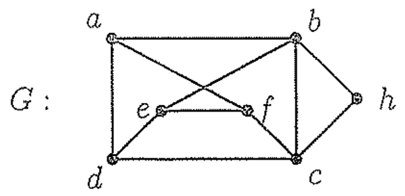
5. The graph G has

(a) a 3-clique as a subgraph;
(b) a 4-clique as a subgraph;
(c) a 4-clique and a 3-clique as subgraphs;
(d) neither a 4-clique nor a 3-clique as subgraphs.

6. The order of every spanning subgraph of G is:

(a) 5; (b) 9; (c) 7; (d) 4.

Questions 7. to 14. refer to the graph G drawn below:



7. The graph G is not a regular graph because:

(a) not all edges are the same length; (b) it is a complete graph;

(c) not all vertices have the same degree; (d) it has a vertex of degree 3.

8. The graph G is a bipartite graph:

(a) TRUE; (b) FALSE.

9. The order in $G - h$ is;

(a) 4; (b) 6; (c) 5; (d) 7.

10. bh is an edge in G. The order in $G - bh$ is:

(a) 4; (b) 6; (c) 5; (d) 7.

11. $dacbeb$ is

(a) a walk in G; (b) a walk but not a trail in G;

(c) a trail but not a path in G; (d) not a walk in G.

12. $befcdef$ is

(a) a trail in G; (b) a walk but not a trail in G;

(c) a cycle in G. (d) a path but not a cycle in G.

$G - ef$ is not disconnected

13. The edge ef is not a bridge of G because:

(a) the degree of e is 3; (b) the edge ef is parallel to the edge dc;

(c) G is 3−regular; (d) $G - ef$ is not connected.

14. The graph G is:

(a) semi-Hamiltonian but not Eulerian;

(b) Hamiltonian but not Eulerian;

(c) Hamiltonian and semi-Eulerian;

(d) not Hamiltonian nor Eulerian.

15. The maximum size of a simple graph of order 15 is

(a) 105; (b) 210; (c) 21; (d) 45.

$H = (V, E)$ is a graph, where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{v_1v_1, v_1v_2, v_5v_3, v_2v_3, v_3v_4, v_4v_3, v_4v_5, v_5v_4\}$.

Questions 16. and 17. refer to $H = (V, E)$.

16. The degree of the vertex v_1 in $H = (V, E)$ is:
(a) 2; (b) 1; (c) 3; (d) 4.

17. The size of $H = (V, E)$ is:
(a) 5; (b) 8; (c) 7; (d) 6.

18. Exactly one of the following sequences is graphic. Which one is it?
(a) $(1, 3, 3, 4, 5, 6, 6)$; (b) $(1, 2, 2, 3, 4, 4)$;
(c) $(2, 3, 3, 4, 4, 5)$; (d) $(2, 3, 4, 4, 5)$.

19. The size of the path graph P_7 is
(a) 5; (b) 6; (c) 4; (d) 7.

20. Let H be a simple graph of order 8 and with 3 components. Then the number of the maximum size of H is:
(a) 15; (b) 10; (c) 11; (d) 24.

21. Let G be the complete graph K_{12}. Then G is: not Eulerian, but Hamiltonian
(a) semi-Eulerian and semi-Hamiltonian; (b) Eulerian and Hamiltonian;
(c) neither Eulerian nor Hamiltonian; (d) Eulerian and not Hamiltonian.

22. The size of the complete bipartite graph $K_{5,4}$ is:
(a) 10; (b) 9; (c) 25; (d) 20.

23. The order of the complete bipartite graph $K_{5,4}$ is:
(a) 10; (b) 9; (c) 25; (d) 20.

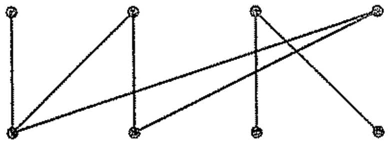
24. The order of the 4–cube graph Q_4 is:
(a) 16; (b) 4; (c) 8; (d) 32.

25. The size of the 4–cube graph Q_4 is:
(a) 16; (b) 4; (c) 8; (d) 32.

26. The order of the Petersen graph is
(a) 16; (b) 4; (c) 10; (d) 6.

27. The size of the complement of the cycle graph C_5 is:
(a) 25; (b) 5; (c) 10; (d) 15.

Questions 28. and 29. refer to the graph G which is drawn below:

G : 

28. The graph G is bipartite
(a) TRUE; (b) FALSE.

29. The graph G has
(a) 2 components; (b) 1 component;
(c) 8 components; (d) 7 components.

30. $H = (V, E)$ is a graph, where $V = \{a, b, c, d, e, f\}$
and $E = \{ab, ad, ac, bc, be, cd, cf, de, df\}$. The edge-set for the complement of H is:
(a) $\{ab, be, de, be, cf, cd\}$ (b) $\{af, ad, ac, bc, be, cd, cf, ae, df\}$
(c) $\{af, fb, bd, dc, ce, ea\}$ (d) $\{ac, fb, ba, dc, fe, ea\}$

edge-set = $\{af, fb, bd, ce, ef, ea\}$

31. State the Hand-Shaking Lemma.

see note p. 24

32. Ore's Theorem states a sufficient condition for a graph to be Hamiltonian.
State Ore's Theorem.

see note p. 103

33. Show that there is no simple graph of order 12 and size 28 in which the degree of each vertex is either 3 or 6.

Suppose we have m vertices of degree 3, n of degree 6.

So $m + n = 12$ (1.)

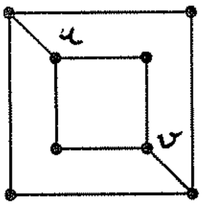
By Hand-Shaking Lemma,

$3m + 6n = 56 = 2 \times 28$ (2.)

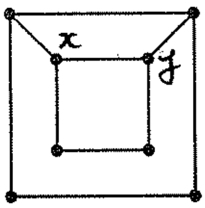
Solving (1.), (2.). $n = \frac{20}{3}$ (not an integer!)

34. Are the graphs G and H drawn below isomorphic? To answer this question, either label the graphs in such a way as to illustrate the isomorphism or, in the box below, give a reason to show that the graphs cannot be isomorphic.

G :



H :



Many reasons why $G \not\cong H$.
e.g. In H, x, y (of degree 3) are adjacent; in G u, v (of degree 3) are not!