The University of Sydney

MATH2969/2069

Graph Theory **Tutorial 2 (Week 9)** 2008

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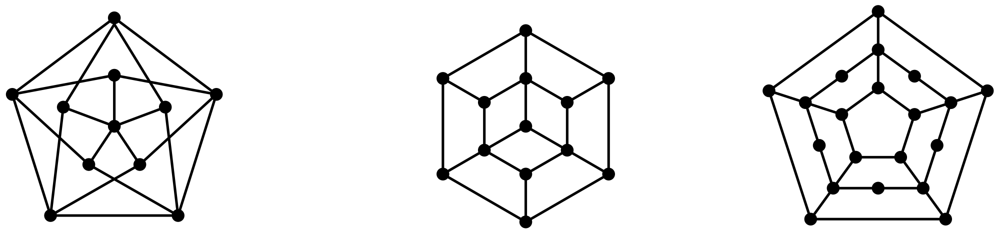
1. Show that the graph on the left is Hamiltonian, but that the other two are not.



2. Is it possible for a simple connected graph containing a bridge to be Hamiltonian?

3. What is wrong with the following argument?

Suppose G is a simple graph with degree sequence $(3, 3, 3, 3, 3, 3, 6)$. Let u and v be two non-adjacent vertices with degree 3, so that $\deg(u) + \deg(v) = 6 \leq 7$. Since the number of vertices is 7, it follows by Ore's Theorem that G is not Hamiltonian.

4. A club has seven members and meets for lunch each month. If the members sit at a round table and decide to sit so that each member has different neighbours at each lunch, determine how many months this arrangement could last, and give possible seating arrangements for these months.

5. (*i*) How many edges does the complete bipartite graph, $K_{m,n}$, have?

(*ii*) How many complete bipartite graphs have k vertices?

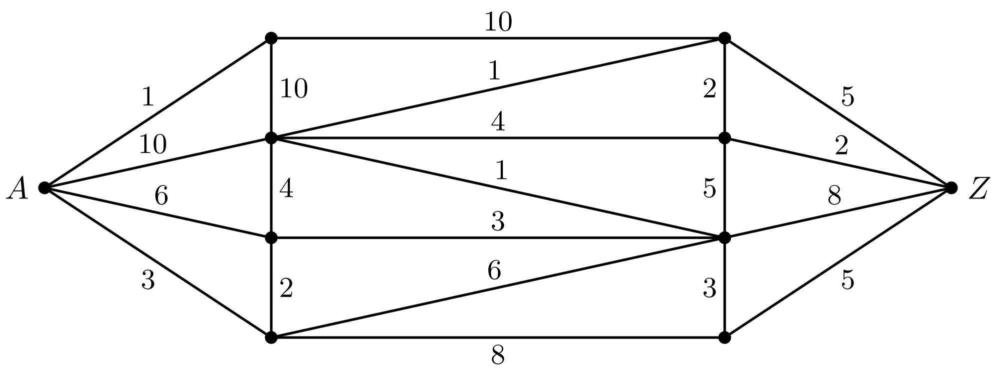
(*iii*) What is the maximum number of edges in a simple bipartite graph with k vertices?

6. (*i*) How many vertices does the k-cube graph, Q_k, have?

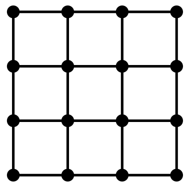
(*ii*) What is the degree of each vertex in Q_k?

(*iii*) How many edges does Q_k have?

7. In the following graph, find all shortest paths from A to Z.



8. Find a solution to the Chinese Postman Problem in this graph, given that every edge has equal weight.



9. (*i*) Prove that a graph in which the degree of each vertex is at least two contains a cycle.

(*ii*) Prove that a tree with at least 2 vertices has at least 2 vertices of degree 1.

10. Prove, directly from the definition of a tree as a connected graph without cycles, that the additon of one edge to a tree creates exactly one cycle.

11. How many edges are there in a forest with v vertices and k components?

12. Let T be a tree with p vertices of degree 1 and q other vertices. Show that the sum of the degrees of the vertices of degree greater than 1 is $p + 2(q - 1)$.

13. Show that if a tree has two vertices of degree 3, then it must have at least 4 vertices of degree 1.

14. Show that, for each value of $n \geq 1$, the graph associated with the alcohol molecule $C_nH_{2n+1}OH$ is a tree. (Carbon, Hydrogen, Oxygen have valencies 4, 1, 2 respectively.)

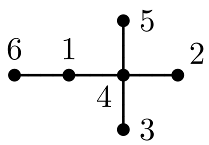
15. (*i*) Find the number of molecules with formula C_5H_{12}, and draw them.

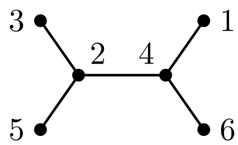
(*ii*) How many non-isomorphic trees are there with 5 vertices?

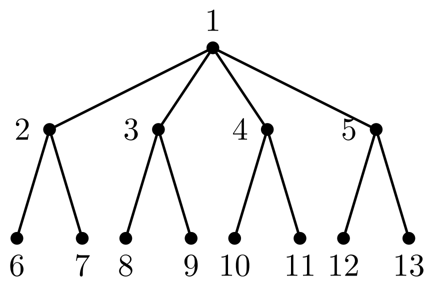
(*iii*) Comment on the relationship between your answers to parts (*i*) and (*ii*).

16. Repeat question **7** for C_6H_{14}, and trees with 6 vertices.

17. Find the Prüfer sequence corresponding to each of the following labelled trees:







18. Find the labelled trees corresponding to these Prüfer sequences:

(*i*) $(1, 2, 3, 4, 5)$ (*ii*) $(3, 3, 3, 3, 3)$ (*iii*) $(2, 8, 6, 3, 1, 2)$