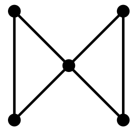
THE UNIVERSITY OF SYDNEY

MATH2969/2069

Graph Theory **Tutorial 3 (Week 10)** 2008

1. Draw all the spanning trees of this graph:



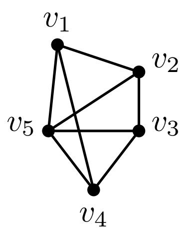
2. If A is the adjacency matrix of a simple graph G with vertex set $\{v_1, v_2, \ldots, v_n\}$, and $\left(A^k\right)_{ij}$ is the (i, j) term of A^k, show that $\left(A^2\right)_{ii} = \deg(v_i)$.

3.

(i) Write down the adjacency matrix, A, for this graph.

(ii) Calculate A^2, and verify that the result proved in Question **2** holds.

(iii) Find the number of different walks of length 4 from v_5 to v_5.



(iv) Verify that the trace of A^3 is 6 times the number of triangles in the graph.

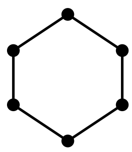
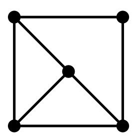
4. A graph G has adjacency matrix $A = \begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$.

(i) Is G a simple graph? (ii) What is the degree sequence of G?

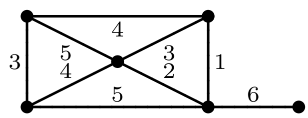
(iii) How many edges does G have?

5. Let A be the adjacency matrix of a bipartite graph. Prove that the diagonal entries of A^{2n+1} are all equal to 0, for any natural number n.

6. Find the number of spanning trees in each of the following graphs:

(i)  (ii) K_7 (iii) $K_{3,3}$ (iv) 

7. Use Kruskal's algorithm to find all least weight spanning trees for this weighted graph.



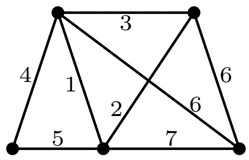
8. Find a minimum cost spanning tree for the graph with this cost matrix. How many such trees are there?

	A	B	C	D	E	F	G	H
A	0	12	0	14	11	0	17	8
B	12	0	9	0	12	15	10	9
C	0	9	0	18	14	31	0	9
D	14	0	18	0	0	6	23	14
E	11	12	14	0	0	15	16	0
F	0	15	31	6	15	0	8	16
G	17	10	0	23	16	8	0	22
H	8	9	9	14	0	16	22	0

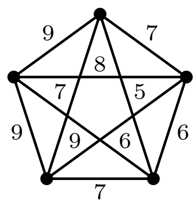
Extra questions

9. Find a minimum weight spanning tree in each of the following weighted graphs:

(i)



(ii)



10. Use the Matrix Tree Theorem to verify the fact that the number of spanning trees of the complete graph K_n is equal to the number of labelled trees on n vertices.