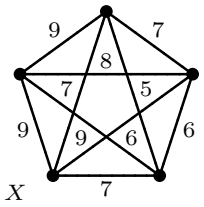
The University of Sydney

MATH2969/2069

**Graph Theory** **Tutorial 4 (Week 11)** 2008

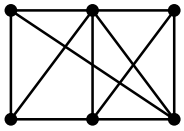
**1.** (*i*) How many Hamiltonian cycles are there in this graph?




(*ii*) Delete the vertex labelled $x$ (and its incident edges). How many spanning trees are there on the remaining subgraph? How many of these spanning trees are part of a Hamiltonian cycle?

(*iii*) Find a minimum weight spanning tree on the remaining subgraph. Hence show that the weight (or length) of a solution to the travelling salesman problem is at least 32.

**2.** Show that this graph is planar by drawing it in the plane without any edges crossing. Verify Euler's formula for this graph.



**3.** (*i*) Show that a connected simple planar graph all of whose vertices have degree at least 5 must have at least 12 vertices.

(*ii*) Show that a connected simple planar graph with fewer than 30 edges has at least one vertex of degree at most 4.

(*iii*) Show that a connected simple planar graph has at least one vertex of degree at most 5.

**4.** Consider a connected simple planar graph with $v$ ($\geq 3$) vertices, $e$ edges and $f$ regions.

(*i*) Show that if $e = 3v - 6$ then each region is a triangle.

(*ii*) Deduce that a convex polyhedron with 12 vertices and 20 faces is composed entirely of triangles.

**5.** A graph is said to be *polyhedral* if it is simple, connected, and planar, and every vertex has degree at least 3.

(*i*) Prove that a polyhedral graph cannot have exactly seven edges.

(*ii*) Prove that no polyhedral graph has 30 edges and 11 regions.

**6.** Give an example of a connected planar graph in which $e > 3v - 6$.

**7.** In this graph, find a subgraph which is homeomorphic to $K_{3,3}$.

Is this graph planar or nonplanar?