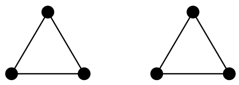
THE UNIVERSITY OF SYDNEY

MATH2969/2069

Graph Theory **Tutorial 5 (Week 12)** 2008

1. (*i*) Let G be the disconnected planar graph shown. Draw its dual G^*, and the dual of the dual $(G^*)^*$.

(*ii*) Show that if G is a disconnected planar graph, then G^* is connected. Deduce that $(G^*)^*$ is not isomorphic to G.

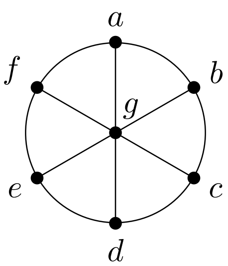


G

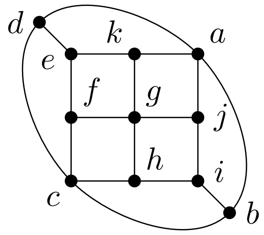
2. A certain polyhedron has faces which are triangles and pentagons, with each triangle surrounded by pentagons and each pentagon surrounded by triangles. If every vertex has the same degree, p say, show that $\frac{1}{e} = \frac{1}{p} - \frac{7}{30}$. Deduce that $p = 4$, and that there are 20 triangles and 12 pentagons. Can you construct such a polyhedron?

State the dual result.

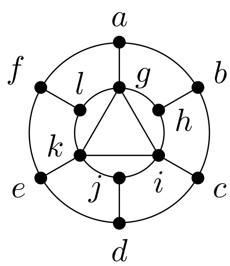
3. Determine the chromatic number of each of the following graphs:



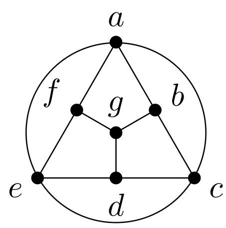
(*i*)



(*ii*)



(*iii*)



(*iv*)

4. For each of the following graphs, what does Brooks' Theorem tell you about the chromatic number of the graph? Find the chromatic number of each graph.

(*i*) The complete graph K_{20}.
(*ii*) The bipartite graph $K_{10,20}$.
(*iii*) A cycle with 20 edges.
(*iv*) A cycle with 29 edges.
(*v*) The cube graph Q_3.
(*vi*) The dual of Q_3.

5. (*i*) Determine the minimum number of colours required to colour the faces of Q_3 in such a way that adjoining faces have a different colour.

(*ii*) Repeat part (*i*) for the dual of Q_3.

6. Show that a simple connected planar graph with 17 edges and 10 vertices cannot be properly coloured with two colours.
(Hint: Show that such a graph must contain a triangle.)

7. Let T be a tree with at least 2 vertices. Prove that $\chi(T)$=2.

8. Hubert keeps five varieties (A, B, C, D, E) of snakes in boxes in his apartment. Some varieties attack other varieties, and can't be kept together. In the table, an asterisk indicates that varieties can't be kept together. What is the minimum number of boxes needed?

	A	B	C	D	E
A	–	*	*	*	*
B	*	–	*	–	–
C	*	*	–	*	–
D	*	–	*	–	*
E	*	–	–	*	–

9. Determine the number of ways in which each of the following graphs can be properly coloured, given λ different colours.

(i) The complete graph K_6. (ii) The star graph $K_{1,5}$.

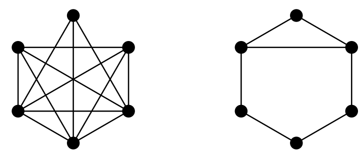
(iii) The linear graph L_6.

10. (i) Find the chromatic polynomials of each of the six connected simple graphs on four vertices.

(ii) Verify that each of the polynomials in (i) has the form $\lambda^4 - e\lambda^3 + a\lambda^2 - b\lambda$ where e is the number of edges and a and b are positive constants.

11. Find the chromatic polynomials of $K_{1,n}$, $K_{2,n}$ and $K_{3,n}$.

12. State two reduction formulas for chromatic polynomials. Use whichever seems appropriate to calculate the chromatic polynomial for each of the two given graphs. Also determine the chromatic number of each graph.

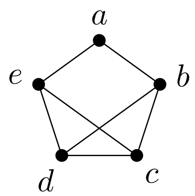


13. Find the chromatic polynomial of C_5, the cycle with 5 vertices.

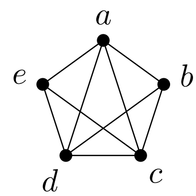
14. Explain why the chromatic polynomial $P_G(\lambda)$ of a planar graph G cannot contain a term $(\lambda - k)$ for any $k \geq 4$.

15. Find the chromatic index (or edge-chromatic number) of the graph G, where G is:

(a)



(b)



16. Find the chromatic index of the cube, and of the octahedron.