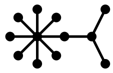
The University of Sydney

MATH2969/2069

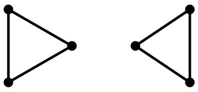
Graph Theory **Tutorial 6 (Week 13)** 2008

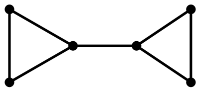
1. Find $\chi(G)$ and $\chi'(G)$ in each of the following cases.

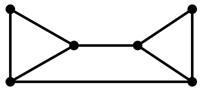
(i) $G = K_9$ (ii) $G = K_{10}$

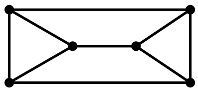
(iii) $G = K_{5,6}$ (iv) $G =$ 

2. Which of the following graphs are orientable?


(a)


(b)


(c)


(d)

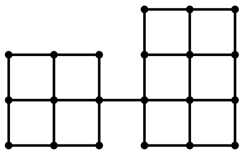
3. Are either of the following statements true?

(i) Any Hamiltonian graph is orientable.

(ii) Any orientable graph is Hamiltonian.

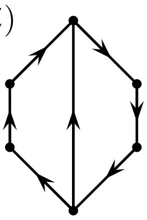
4. Perform a breadth first search, and a depth first search on the Petersen graph. Find a strongly connected orientation for the Petersen graph.

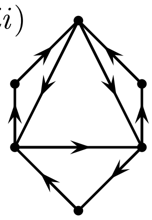
5. Explain why the addition of exactly one edge will make this graph orientable. Add the edge, and then find a strongly connected orientation.

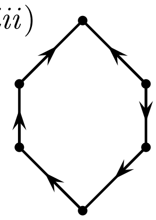


6. Determine which of the following graphs are

(a) strongly connected; (b) Eulerian; (c) Hamiltonian.

(i)


(ii)


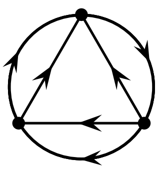
(iii)


7. (i) Is this digraph simple?

(ii) Is it Eulerian?

(iii) Is it Hamiltonian?

(iv) Is it strongly connected?



8. In a tournament, the *score* of a vertex is its out-degree, and the *score sequence* is a list of all the scores in non-decreasing order.

(*i*) If the score sequence of a tournament is $(s_1, s_2, \dots, s_n)$, show that $\sum_{i=1}^{n} s_i = \frac{n(n-1)}{2}$.

(*ii*) What is the score sequence of a tournament with n vertices if each vertex has a different score?

(*iii*) Find a semi-Hamiltonian path in a tournament with n vertices $v_0, v_1, \dots, v_{n-1}$ if the score of vertex v_i is i.