

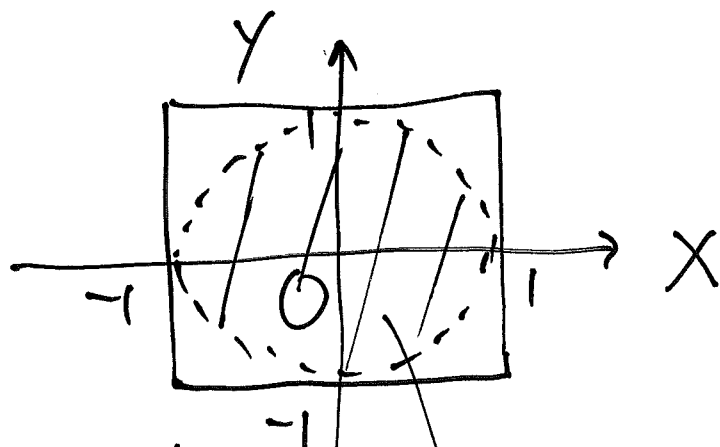
Monte-Carlo Methods

Eg 7.4 Estimate π

Area of a circle of unit radius = π

Suppose we randomly choose a point (X, Y) in the square

$$[-1, 1] \times [-1, 1], \text{ i.e. } \begin{cases} -1 \leq x \leq 1 \\ -1 \leq y \leq 1 \end{cases}$$



unit circle
area = π

area of square = 4

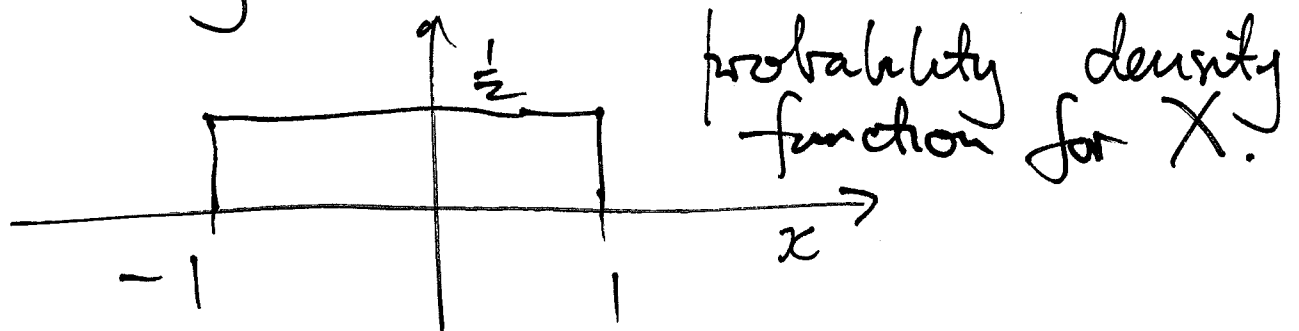
The probability that the point (X, Y) falls in the circle is the ratio of the areas, i.e. of the

circle to the square, i.e. $\pi/4$

$$\text{Thus } P(X^2 + Y^2 < 1) = \frac{\text{area of circle}}{\text{area of square}} = \pi/4$$

$$\text{So } \pi = 4 P(X^2 + Y^2 \leq 1)$$

The variables X, Y are uniformly distributed on $[-1, 1]$.



X, Y are independent.

We estimate $P(X^2 + Y^2 \leq 1)$ by selecting (X, Y) from a uniform distribution on $[-1, 1] \times [-1, 1]$ and count the frequency that $X^2 + Y^2 \leq 1$, i.e. (X, Y) lies in the circle. If

$$M = \# \text{ of times } X^2 + Y^2 \leq 1$$

$$N = \# \text{ of trials}$$

$$\text{Then } \pi = 4 P(X^2 + Y^2 \leq 1) \approx \frac{4M}{N}.$$