$\underline{\underline{L}}\,\underline{\underline{U}}$ decomposition solution of $\underline{\underline{A}}\,\underline{x} = \underline{b}$

$\underline{\underline{A}}\,\underline{x} = \underline{b}$ $\boxed{\underline{\underline{A}} = \underline{\underline{L}}\,\underline{\underline{U}}}$

$\underline{\underline{L}}\,\underline{\underline{U}}\,\underline{x} = \underline{b}$

Set $\underline{y} = \underline{\underline{U}}\,\underline{x}$

Then

$$\underline{\underline{L}}\,\underline{y} = \underline{\underline{L}}\,\underline{\underline{U}}\,\underline{x} = \underline{b}$$

Solve $\boxed{\underline{\underline{L}}\,\underline{y} = \underline{b}}$ $\rightarrow$ lower-triangular system of equations

for $\underline{y}$

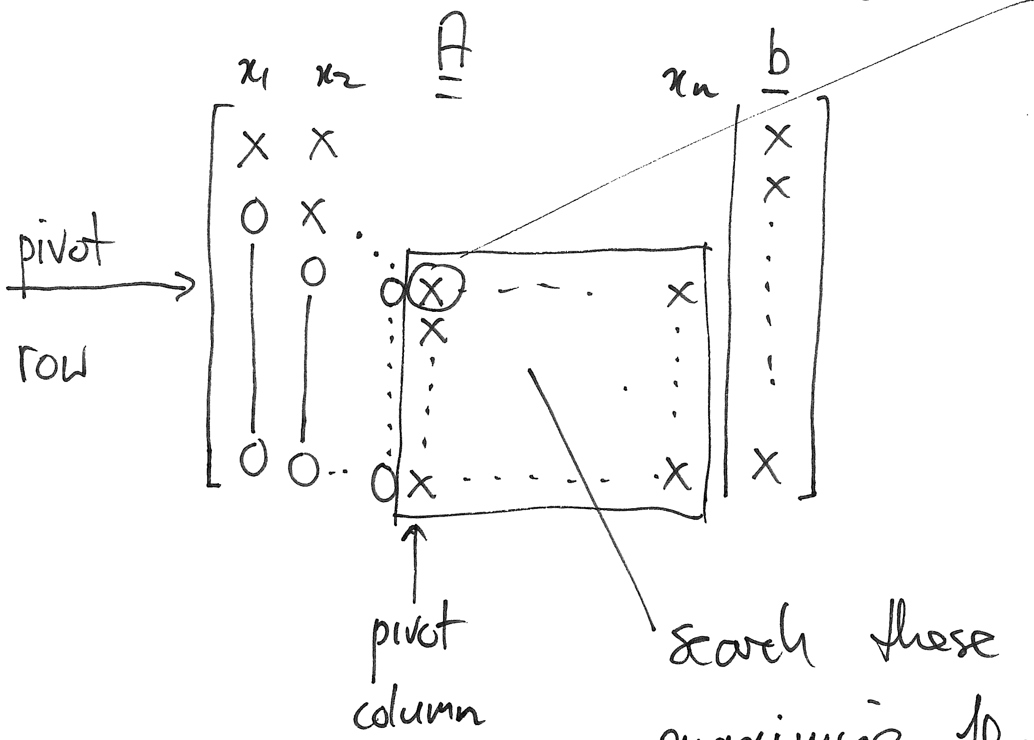
Solve $\boxed{\underline{\underline{U}}\,\underline{x} = \underline{y}}$ $\rightarrow$ upper-triangular system.

for $\underline{x}$.

Very useful for solving many systems with same $\underline{\underline{A}}$ different $\underline{b}$.

Full / Complete Pivoting

current pivot



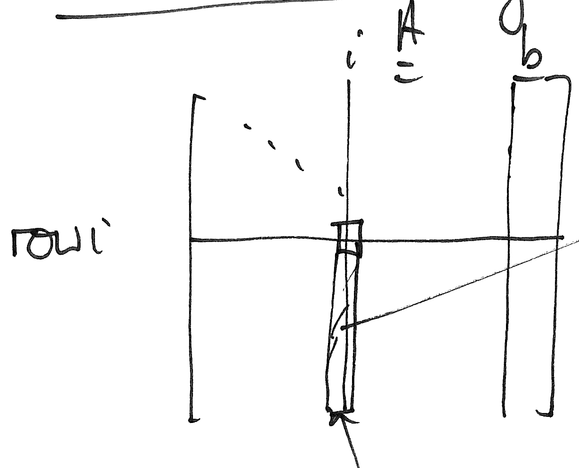
$\underline{\underline{A}}\,\underline{x} = \underline{b}$

search these elements to maximize the pivot (in absolute value).

Drawbacks: search for maximum takes time
column interchanges must be tracked.

Column & row interchanges are used.

Partial Pivoting



Use row interchanges to swap pivot for largest element among these.

pivot chosen from these elements

Used in practice: gepp.f90 does this.

In Eg 4.2

$$\underline{\underline{L}} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{pmatrix} \qquad \underset{\sim}{b} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \begin{aligned} y_1 &= 5 \\ y_2 &= 3 - 2y_1 = -7 \\ y_3 &= -1 - y_1 - 3y_2 = 15 \end{aligned}$$

$$\Rightarrow \quad \underset{\sim}{y} = \begin{pmatrix} \cancel{4}5 \\ -7 \\ 15 \end{pmatrix}$$