

Convergence of Iterative Methods (A)

$$\underline{\delta x}^{(k+1)} = \underline{H} \underline{\delta x}^{(k)}$$

$$\underline{\delta x}^{(k)} = \underline{H} \underline{\delta x}^{(k-1)}$$

⋮

$$\underline{\delta x}^{(1)} = \underline{H} \underline{\delta x}^{(0)}$$

$$\Rightarrow \underline{\delta x}^{(k+1)} = \underline{H}^{k+1} \underline{\delta x}^{(0)}$$

$$\|\underline{\delta x}^{(k+1)}\| = \|\underline{H}^{k+1} \underline{\delta x}^{(0)}\|$$

$$\leq \|\underline{H}^{k+1}\| \|\underline{\delta x}^{(0)}\|$$

$$\leq \|\underline{H}\|^{k+1} \|\underline{\delta x}^{(0)}\|$$

If $\|\underline{H}\| < 1$, then $\|\underline{H}\|^{k+1} \rightarrow 0$ as $k \rightarrow \infty$.

So $\|\underline{\delta x}^{(k+1)}\| \rightarrow 0$ as $k \rightarrow \infty \Rightarrow$

$\underline{\delta x}^{(k+1)} \rightarrow 0$ as $k \rightarrow \infty \Rightarrow$

$\underline{x}^{(k+1)} \rightarrow \underline{x}$ as $k \rightarrow \infty$, if $\|\underline{H}\| < 1$.