

Some Other Operation Counts

1) $n \times 1$ column vectors x, y :

$$\begin{aligned} \text{operation count for } x^T y &= \\ &= nM + (n-1)A \end{aligned}$$

2) $n \times n$ matrix A ; $n \times 1$ column vector x :

$$\begin{aligned} \text{operation count for } Ax &= \\ n [nM + (n-1)A] &= n^2M + n(n-1)A \end{aligned}$$

3) $n \times n$ matrices A, B :

$$\begin{aligned} \text{operation count for } AB &= \\ n [n^2M + n(n-1)A] &= n^3M + n^2(n-1)A \end{aligned}$$

Associativity of Matrix Multiplication

$$A \quad n \times n$$

$$B \quad n \times n$$

$$x \quad n \times 1$$

$$(AB)x = A(Bx) \quad \text{BUT}$$

consider operation counts :

$$AB \quad n^3 M + n^2(n-1) A$$

$$\underline{(AB)x \quad n^2 M + n(n-1) A}$$

$$\text{Total : } (n^3 + n^2) M + n(n+1)(n-1) A$$

$$Bx \quad n^2 M + n(n-1) A$$

$$A(Bx) \quad n^2 M + n(n-1) A$$

$$\text{Total : } 2n^2 M + 2n(n-1) A$$

Which would you do, $(AB)x$ or $A(Bx)$?