The Class of Structurally Regular Semigroups

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1 Abstract

A semigroup S is said to be *structurally regular* if there exists an ordered pair (n, m) of non-negative integers such that the quotient $S/\theta(n, m)$ is regular in the usual sense. The congruence $\theta(n, m)$ may be defined in the following manner:

$$\theta(n, m) = \{(a, b) : uav = ubv, \forall u \in S^n, v \in S^m\},$$

where S^k for $k \geq 1$ denotes the set of all products of k elements in S, while S^0 denotes the singleton set containing only the identity element of the monoid $S^{(1)}$ obtained by adjoining an identity element to S. Alternatively, a semigroup S is structurally regular if there exists some ordered pair (n, m) such that for each element $a \in S$ there exists an element $b \in S$ (not necessarily unique) such that for all $u \in S^n$ and for all $v \in S^m$ the two equalities $uabav = vav$ and $ubabv = ubv$ hold.

The concept of structural regularity was first introduced in [2], and is a generalization of the usual concept of regularity, but is distinct from other well known generalizations such as the classes of: *eventually regular semigroups*, *nilpotent extensions of regular semigroups* or *locally regular semigroups*. Moreover, the class of structurally regular semigroups manifests some interesting resemblance of the class of regular semigroups. In this talk we will present a characterization for semigroups which are structurally inverse

and which are also nilpotent extensions of inverse semigroups. We will also present some interesting open problems.

References

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