

Last time: integrating in polar coordinates

The equation $r = \cos 2\theta$ traces out a “four-leafed rose” as θ varies. In particular, a leaf or petal of the rose is the region enclosed one loop of the curve, given by

$$D = \{(r, \theta) \mid \frac{-\pi}{4} \leq \theta \leq \frac{\pi}{4}, 0 \leq r \leq \cos 2\theta\}.$$

Sketch the curve. Which formula can be used to calculate the area of this leaf?

- (a) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\cos 2\theta} r dr d\theta$
- (b) $2 \int_0^{\frac{\pi}{4}} \int_0^{\cos 2\theta} r dr d\theta$
- (c) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\cos 2\theta} r dr d\theta$
- (d) $\int_0^{\cos 2\theta} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} r d\theta dr$
- (e) I don't know.

Practice with triple integrals

We wrote

$$E = \{(x, y, z) \mid (x, z) \in D_2, x^2 + z^2 \leq y \leq 4\},$$

where $D_2 = \{(x, z) \mid x^2 + z^2 \leq 4\}$. so

$$\begin{aligned} V(E) &= \iiint_E dV \\ &= \iint_{D_2} \int_{x^2+z^2}^4 dy \, dA \\ &= \iint_{D_2} [y]_{x^2+z^2}^4 dA \\ &= \iint_{D_2} 4 - x^2 - z^2 dA. \end{aligned}$$

Practice with triple integrals

So we need to calculate $\iint_{D_2} 4 - x^2 - z^2 dA$. We could write D_2 as a region of type I or II, but it is easier to use polar coordinates (in the xz -plane):

$$x = r \cos \theta, \quad z = r \sin \theta;$$

$$\begin{aligned} \iint_{D_2} 4 - x^2 - z^2 dA &= \int_0^{2\pi} \int_0^2 (4 - r^2) r \, dr d\theta \\ &= \int_0^{2\pi} \left[2r^2 - \frac{1}{4}r^4 \right]_0^2 d\theta \\ &= \int_0^{2\pi} 4 d\theta = 8\pi. \end{aligned}$$

Practice with triple integrals

E is the region bounded by the planes $x = 2$, $y = 2$, $z = 0$ and $x + y - 2z = 2$, and D is its triangular shadow in the xy -plane.

Fill in the blanks, and discuss with your neighbour:

$$D = \{(x, y) \mid ? \leq x \leq ?, ?? \leq y \leq ??\}.$$

- (a) We're working on it.
- (b) We're stuck.
- (c) We have answers, but they're different and we don't know who is right.
- (d) We have the same answer.

Practice with triple integrals

Now find $u_1(x, y)$ and $u_2(x, y)$ such that

$$E = \{(x, y, z) \mid (x, y) \in D, u_1(x, y) \leq z \leq u_2(x, y)\}.$$

- (a) We're working on it.
- (b) We're stuck.
- (c) We have answers, but they're different and we don't know who is right.
- (d) We have the same answer.