AN INTRODUCTION TO BRAID GROUPS AND THEIR CATEGORICAL ACTIONS

T. GOBET AND G. WILLIAMSON

The theory of braid groups occupies a beautiful place in modern mathematics. The theory began with Artin in an attempt to understand knots; nowadays braids play an important role in topology, representation theory and symplectic geometry. This course will be an introduction to the theory with an emphasis on current research directions. We will cover

(1) The basic theory of Coxeter groups (Exchange property, word problem) and finite reflection groups ([6]).
(2) The various topological realizations of the braid group of type A and the theory of braid automorphisms (leading to Artin's solution to the word problem [1], [7]).
(3) The basic theory of Artin-Tits groups attached to arbitrary Coxeter groups (definition by generators and relations, Garside structure when the Coxeter group is finite [3]) and state the main open problems (linearity, word problem, $K(\pi,1)$-conjecture, center, torsion [4]). See [5] for a survey of the basic questions and techniques used in the study of braid groups.
(4) Some topics in the representation theory of braid groups of type A, including Hecke algebras, the Burau representation [7] and the Lawrence-Krammer-Bigelow representation [9] (a faithful representation of Artin's braid group).
(5) The recent theory of categorical actions of Artin-Tits groups, discussing in detail the work of Khovanov-Seidel [8] and Brav-Thomas [2]. If time allows, we will also discuss Rouquier's [10] approach to faithful categorical actions of arbitrary Artin-Tits groups.
(6) Affine braid groups [11].

Please note that the bibliography will be updated during the semester.

References

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