Solitons: Ball-Box Model Approach

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Summary

In this study, we briefly discuss the behaviour of multi-soliton solutions for the discretised Korteweg-de Vries (KdV) equation. We review the solution of the KdV equation for one soliton. We then found exact one-, two- and three-soliton solutions of the KdV equation in the bilinear form by applying Hirota's bilinear D-operator. We examine the asymptotic properties of the two-soliton solution of the KdV equation as $t \to \pm\infty$ to study the phase shift during the collision of the solitons. The Ball-Box Model is also discussed.

Solitons and their properties

A soliton is a special kind of solitary wave which is obtained as the solution of a nonlinear integrable PDE. The following are the properties of solitons:

- They are localised.
- They keep their localised form over time.
- They are preserved after collision with one another.

Kortweg-De Vries (KDV) Equation

The equation is of the form:

$$u_t + 6uu_x + u_{xxx} = 0.$$

With the boundary conditions u,u_x,$u_{xxx} \to 0$ as $x \to \pm\infty$.

- Direct integration:

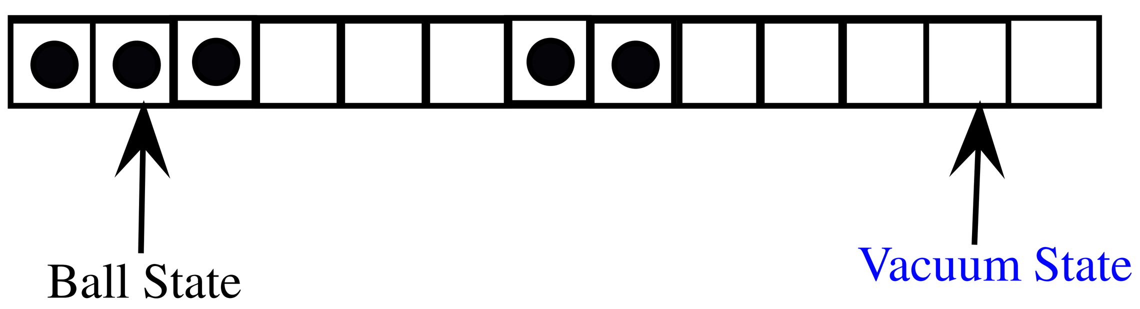
$$u(x,t) = \frac{\beta}{2} sech^2\left(\frac{\sqrt{\beta}}{2}(x - \beta t)\right).$$

- Hirota bilinear method:

$$u(x,t) = \frac{a^2}{2} sech^2\left(\frac{1}{2}(ax - a^3 t - c)\right),$$

Ball-box model (BBM)

The BBM is used to describe the behaviour of multiple-soliton solutions. A state of this model consists of an infinite array of boxes with a finite number of balls evolving in time such that each box contains at most one ball.



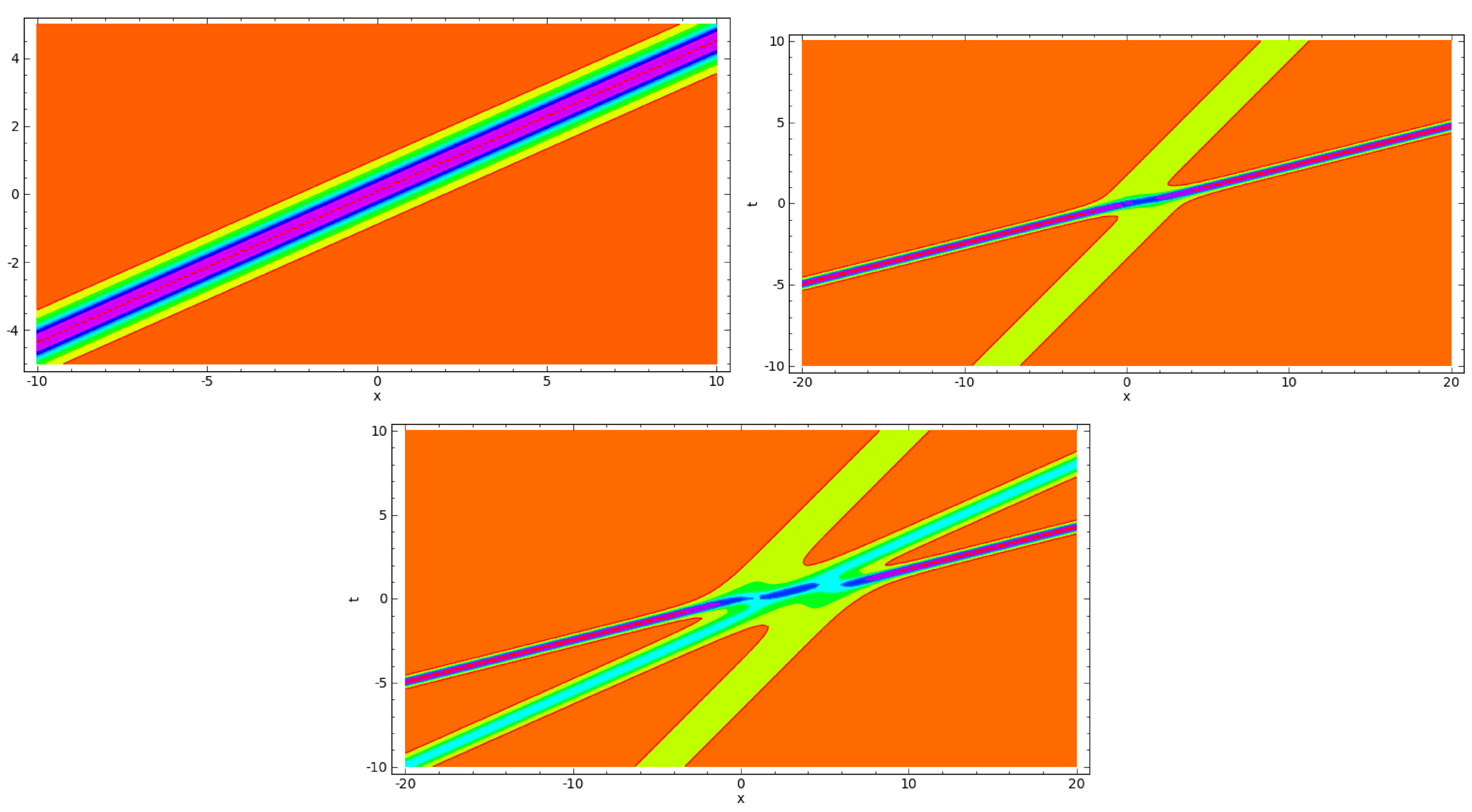
Examples

Examples of PDEs that admit soliton solution are

- Korteweg-de Vries (KdV) equation
- Sine-Gordon equation
- nonlinear Schrödinger equation

Numerical simulations

The contour plots of one-, two- and three-solitons:



- Phase shift of slow moving soliton:

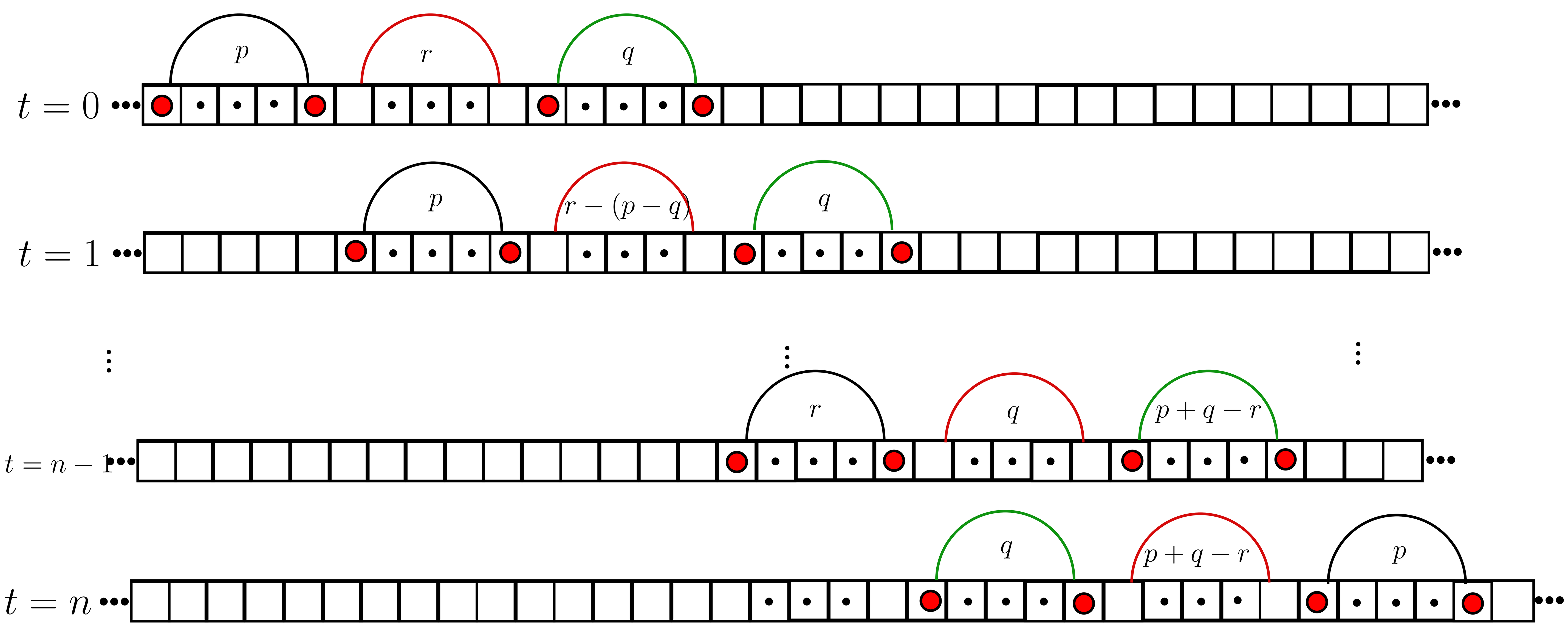
$$-2\log\left(\frac{a_1 - a_2}{a_1 + a_2}\right),$$

- Phase shift of fast moving soliton:

$$2\log\left(\frac{a_1 - a_2}{a_1 + a_2}\right),$$

General Case of BBM

An example of a case with the initial separation $r > p$, applying the time evolution rule we have:



The phase shift for the size-p soliton is $\phi = 2q$ while that of size-q soliton is $\phi = -2q$.