THE UNIVERSITY OF SYDNEY

FACULTIES OF ARTS AND SCIENCE

**STAT 2004: Hypothesis Testing**

Semester 2, 2004 Time allowed: Two hours

There are five questions
All questions may be attempted
Questions are NOT of equal value
Non-programmable electronic calculators may be used
Statistical tables will be provided

1. [**15 marks**] The following reaction times (measured to the nearest second) were recorded for 10 drivers both before and after having a controlled amount of alcohol.

| Driver | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| before | 3 | 6 | 5 | 7 | 6 | 4 | 2 | 5 | 7 | 5 |
| after | 3 | 7 | 5 | 6 | 8 | 4 | 4 | 9 | 4 | 6 |

   (a) Test the claim that the mean reaction time has increased after having a controlled amount of alcohol by using a $t$-test (State formally the hypothesis being tested and the evidence against it).

   (b) Test the same hypothesis as in (a) by using the Wilcoxon signed-rank test.

   (c) Compare the assumptions required for the tests in (a) and (b).

2. [**15 marks**] It has been hypothesized that treatments (after casting) of a plastic used in optic lenses will improve wear. Four treatments are to be tested. To determine whether any differences in mean wear exist among treatments, 40 castings from a single formulation of the plastic were made and 10 castings were randomly assigned to each of the treatments. Wear was determined by measuring the increase in "haze" after 200 cycles of abrasion (better wear being indicated by smaller increase). The data collected are reported in the accompanying table.

| Treatment | A | B | C | D |
|---|---|---|---|---|
| | 64 | 65 | 69 | 72 |
| | 62 | 62 | 72 | 70 |
| | 66 | 68 | 71 | 72 |
| | 65 | 67 | 66 | 69 |
| | 60 | 67 | 76 | 73 |
| | 61 | 63 | 74 | 71 |
| | 65 | 67 | 71 | 72 |
| | 66 | 66 | 66 | 68 |
| | 65 | 63 | 68 | 68 |
| | 63 | 72 | 67 | 71 |
| Average | 63.7 | 66 | 70 | 70.6 |

$$\sum\sum x_{ij} = 2703, \qquad \sum\sum x_{ij}^2 = 183231.$$

(a) Construct a ANOVA table and show if there is evidence of a difference in mean wear among the four treatments.

(b) Test if there is a difference between the treatments $B$ and $D$.

(c) Describe the statistical model underlying the analysis and indicate the hypotheses used in (a) and (b).

3. [**15 marks**] From time to time, one branch office of a company must make shipments to another branch office in another state. Three package delivery services operate between the two cities where the branch offices are located. Because the price structure for the three delivery services are quite similar, the company wants to compare the delivery times. The company plans to make several different types of shipments to its branch office. To compare the carriers, the company sends each shipment in triplicate, one with each carrier. The results listed in the accompanying table are the delivery times in hours.

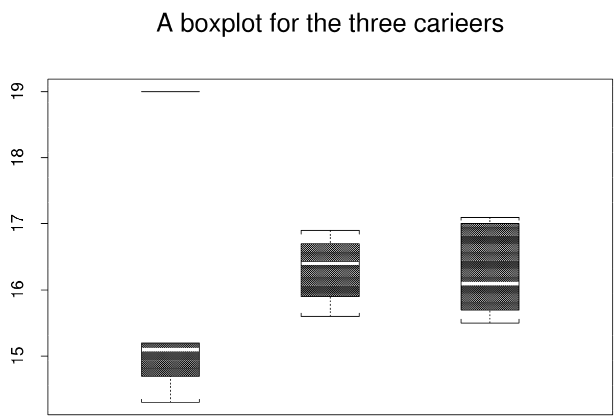
| | Carrier | | | |
|---|---|---|---|---|
| Shipment | I | II | III | Average |
| 1 | 15.2 | 16.9 | 17.1 | 16.4 |
| 2 | 14.3 | 16.4 | 16.1 | 15.6 |
| 3 | 14.7 | 15.9 | 15.7 | 15.4 |
| 4 | 15.1 | 16.7 | 17.0 | 16.3 |
| 5 | 19.0 | 15.6 | 15.5 | 16.7 |
| Average | 15.66 | 16.30 | 16.28 | |

$$\sum\sum x_{ij} = 241.2, \qquad \sum\sum x_{ij}^2 = 3897.62.$$

(a) Construct a two-way ANOVA table and investigate whether there is evidence of a difference in mean delivery times among the three carriers.

(b) Use the Friedman test to see whether there is evidence of a difference in mean delivery times among the three carriers.

(c) Comment the claim that the Friedman test is more reliable based on the basis of the following boxplot.



4. [**10 marks**] You are informed that $X_1, X_2, \cdots, X_{100}$ is a sample from a normal distribution $X \sim N(\mu, 5^2)$, and a test of the hypothesis

$$H: \ \mu = 5 \qquad \text{vs} \qquad H_A: \ \mu > 5$$

is given by

$$\text{Reject } H \text{ if } \bar{x} \geq 5.8.$$

(a) Find the significance level of this test.

(b) Find the power of this test at the alternative value $\mu_a = 7$.

5. [**10 marks**] A study was conducted to determine whether a linear relationship exists between the breaking strength $y$ of wooden beams and the specific gravity $x$ of the wood. The randomly selected beams of the same cross-sectional dimensions were stressed until they broke. The breaking strengths and the density of the wood are shown in the accompanying table for each of the ten beams.

| Beam | Specific Gravity ($x$) | Strength ($y$) |
|---|---|---|
| 1 | 1.00 | 1.1 |
| 2 | 0.95 | 1.7 |
| 3 | 0.94 | 2.0 |
| 4 | 0.90 | 2.4 |
| 5 | 0.86 | 2.9 |
| 6 | 0.83 | 3.2 |
| 7 | 0.75 | 4.0 |
| 8 | 0.62 | 4.3 |
| 9 | 0.55 | 5.1 |
| 10 | 0.40 | 5.5 |

$$\sum y_i = 32.2, \quad \sum x_i = 7.8, \quad \sum y_i^2 = 123.26, \quad \sum x_i^2 = 6.434, \quad \sum x_i y_i = 22.578.$$

(a) Fit the model $Y = \alpha + \beta x + \epsilon$

(b) Test $H: \ \beta = 0$ against $H_A: \beta \neq 0$, where $\epsilon \sim N(0, \sigma^2)$

(c) Find the mean strength for beams with specific gravity 0.78, and give a 90% confidence interval.

Formulae sheet for STAT2004 examination

- $S_{xx} = \sum_i x_i^2 - n(\bar{x})^2$ and $S_{xy} = \sum_i x_i y_i - n(\bar{x})(\bar{y})$.
- $w = \min\{w^+, w^-\}, \quad w^+ = \sum_{j:\, x_j - \mu_0 > 0} r_j, \quad w^- = \sum_{j:\, x_j - \mu_0 < 0} r_j,$

$$EW^+ = \frac{1}{2} \sum_{j:\, x_j - \mu_0 \neq 0} r_j, \quad var(W^+) = \frac{1}{4} \sum_{j:\, x_j - \mu_0 \neq 0} r_j^2.$$

- (One way ANOVA) $\bar{x}_{i.} = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij}, \bar{x} = \frac{1}{N} \sum_{i=1}^{g} \sum_{j=1}^{n_i} x_{ij}$ and $CM = N(\bar{x})^2$,

$$\text{Total SS} = \sum_{i=1}^{g} \sum_{j=1}^{n_i} x_{ij}^2 - CM, \quad \text{Group SS} = \sum_{i=1}^{g} n_i (\bar{x}_{i.})^2 - CM.$$

Individual comparison:

$$t_{lm} = \frac{\bar{x}_{l.} - \bar{x}_{m.}}{s\sqrt{\frac{1}{n_l} + \frac{1}{n_m}}}, \quad \text{where } s^2 = \frac{\text{Residual SS}}{N-g}.$$

- (Two-way ANOVA) $\bar{x}_{i.} = \frac{1}{s} \sum_{j=1}^{s} x_{ij}, \quad \bar{x}_{.j} = \frac{1}{r} \sum_{i=1}^{r} x_{ij}, \ \bar{x} = \frac{1}{rs} \sum_{i=1}^{r} \sum_{j=1}^{s} x_{ij}$ and $CM = rs(\bar{x})^2$;

$$\begin{aligned} \text{Total SS} &= \sum_{i=1}^{r} \sum_{j=1}^{s} x_{ij}^2 - CM; \\ \text{Block SS} &= s \sum_{i=1}^{r} (\bar{x}_{i.})^2 - CM; \\ \text{Treatment SS} &= r \sum_{j=1}^{s} (\bar{x}_{.j})^2 - CM; \end{aligned}$$

The Friedman test: Let $r_{ij}$ denotes the rank of $x_{ij}$ in the $i$-th Block.

$$\bar{r}_{\cdot j} = \frac{1}{r} \sum_{i=1}^{r} r_{ij}, \quad \bar{r} = \frac{1+s}{2}, \quad Q = \frac{r \sum_{j=1}^{s} (\bar{r}_{\cdot j} - \bar{r})^2}{\frac{1}{r(s-1)} \sum_{i=1}^{r} \sum_{j=1}^{s} (r_{ij} - \bar{r})^2}$$

In the case of no ties,

$$Q = \frac{12r}{s(s+1)} \sum_{j=1}^{s} (\bar{r}_{\cdot j})^2 - 3r(s+1).$$

- Regression ANOVA table

| Source | df | SS | MS | F |
|---|---|---|---|---|
| Regression | 1 | $S_{xy}^2/S_{xx}$ | RMS | $\frac{\text{RMS}}{s^2}$ |
| Residuals | $n-2$ | $S_{yy} - S_{xy}^2/S_{xx}$ | $s^2$ | |
| Total | $n-1$ | $S_{yy}$ | | |

$$\begin{aligned} \hat{\beta} &= S_{xy}/S_{xx}, \qquad \hat{\alpha} = \bar{y} - \hat{\beta}\,\bar{x} \\ T_n &= \frac{\hat{\beta}}{s/\sqrt{S_{xx}}}, \quad F_n = \frac{\hat{\beta}^2}{s^2/S_{xx}}, \quad s^2 = \frac{S_{yy} - S_{xy}^2/S_{xx}}{n-2}. \end{aligned}$$

Confidence Interval:

$$\left[\hat{\theta} - t_{\alpha/2} s \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}},\ \hat{\theta} + t_{\alpha/2} s \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}\right]$$

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