CONFIDENTIAL

8112

SURNAME: .... OTHER NAMES: ....

SID:... SEAT NUMBER:.........................

THE UNIVERSITY OF SYDNEY

FACULTIES OF ARTS AND SCIENCE

STAT2012: STATISTICAL TESTS

November 2005 - Time allowed: Three hours and 10 minutes reading

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Instructions to Candidates

This examination contains **10** multiple choice questions and **5** extended answer questions.

Attempt all questions. Statistical tables are provided. Your own Non-Programmable calculator may be used.

In Section I, mark your best choice clearly. In Section II, show your working clearly and neatly.

One A4 sheet of handwritten notes may be taken in to the examination.

Section I - Multiple Choice Questions - 2 marks each

1. Let $X_1, \cdots, X_{10}$ be a random sample from an $N(12, 8.1)$ distribution and let $T = X_1 + \cdots + X_{10}$ be the sample total. $P(T > 129)$ is:

   (a) 0.1587 (b) 0.8413 (c) 0.8643 (d) 0.0228 (e) 0.6915

2. In a study of the effect of oral contraceptives (OCs) on blood pressure, measurements were made on 10 OC users (X) and 14 non-users (Y). It is decided to use the two-sample t test to test $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X \neq \mu_Y$ using the statistic $T = \frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{S_p\sqrt{1/n_X+1/n_Y}}$. Based on the sample information, it is found that $T_{obs} = 1.59$. The P-value of the test is:

   (a) $2P(t_{24} \geq 1.59)$ (b) $P(t_{22} \geq 1.59)$ (c) $2P(t_{22} \leq 1.59)$ (d) $2P(t_{22} \geq 1.59)$
   (e) $P(t_{22} \leq 1.59)$

3. Suppose that we wish to test the null hypothesis $H_0 : \mu = 10$ (the mean) for a normal population with $\sigma^2 = 16$ against $H_1 : \mu > 10$. The value of K such that $\bar{x} > K$ provides a critical region of size $\alpha = 0.05$ for a random sample of 25 is:

   a) 10.316 (b) 11.316 (c) 11.645 (d) 1.645 (e) 10.05.

4. A random sample of size $n = 22$ from a normal population with the variance $\sigma^2 = 196$ gave the mean $\bar{x} = 62.8$. A 95% confidence interval (CI) for the population mean μ has length:

   (a) $2 * \frac{1.96*14}{22}$ (b) $\frac{1.96*196}{22}$ (c) $\frac{1.96*14}{\sqrt{22}}$ (d) $\frac{1.96*14}{\sqrt{22}}$ (e) $2 * \frac{1.96*14}{\sqrt{22}}$

5. A manufacturer of refrigerators receives supplies of metal plates supposedly of uniform thickness. In a random sample of 12 plates randomly taken from a large shipment, the standard deviation of thickness is $s = 0.2226$. A 90% confidence interval (CI) for the population variance σ^2 is:

   (a) $(\frac{12*0.2226^2}{21.026}, \frac{12*0.2226^2}{5.226})$ (b) $(\frac{11*0.2226^2}{19.675}, \frac{11*0.2226^2}{4.575})$ (c) $(\frac{11*0.2226}{19.675}, \frac{11*0.2226}{4.575})$

   (d) $(\frac{12*0.2226}{19.675}, \frac{12*0.2226}{4.575})$ (e) $(\frac{12*0.2226^2}{19.675}, \frac{12*0.2226^2}{4.575})$

6. An archeologist is interested of testing $H_0 : \sigma_A^2 = \sigma_B^2$ against $H_0 : \sigma_A^2 \neq \sigma_B^2$ in order to compare the variability of the width of skulls (cranial widths) for skulls found at two different sites A and B. A sample of size 8 from the site A gave the standard deviation, $s_A = 0.388$ and a sample of size 5 from the site B gave the standard deviation, $s_B = 0.555$. Assuming the two samples come from two independent normal populations, the P-value for this test is:

   (a) more than 0.05 (b) less than 0.05 (c) equal to 0.05

   (d) $0.388^2/0.555^2 = 0.4887396$ (e) can not estimate

7. A random sample of size 64 is to be used to test the null hypothesis, $H_0 : \mu \leq 40$ against $H_1 : \mu > 40$ for a population from $N \sim (\mu, 16^2)$. The null hypothesis is rejected if and only if the sample mean, $\bar{x} > 43$. The probability of type I error when $\mu = 37$ is:

   (a) 0.9332 (b) 0.9987 (c) 0.0013 (d) 0.0668 (e) 0.05

8. A group of 150 executives were classified in the table below by sex and whether or not they were firstborn.

Firstborn

	Yes	No	Total
Male	34	74	108
Female	20	22	42
Total	54	96	150

Under the null hypothesis that the two variables Sex and Firstborn are independent, the observed value of $X_{obs}^2 = 3.42$. The P-value of the test is given by:

   (a) $P(\chi_1^2 \geq 3.42)$ (b) $P(\chi_2^2 \geq 3.42)$ (c) $P(\chi_4^2 \geq 3.42)$ (d) $P(Z \geq 3.42)$
   (e) $P(t_4 \geq 3.42)$.

9. A subcommittee consisting of two professors is to decide which of five students is to receive the prestigious Bhargi award. The professors rank the five as follows:

Student	Professor U	Professor V
B.Patin	3	1
M.Tin	4	4
I.Dong	5	5
G.Pong	2	3
H.And	1	2

The Spearman's rank correlation coefficient for these ranks is

(a) 0.3 (b) -0.7 (c) 0.9 (d) 1.0 (e) 0.7

10. A multiple linear regression model in the form of $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ has been fitted to a set of data using the Splus. Below we produce a part Splus output.

```
> l=lm(y~x1+x2+x3)
>summary(l)
Call: lm(formula = y ~ x1 + x2 + x3)
Coefficients:
                  Value Std. Error   t value   Pr(>|t|)
(Intercept)   2097.3950   913.6515    2.2956     0.0356
         x1      6.3356     3.7062    1.7095     0.1067
         x2     12.8548     5.8247    2.2070     0.0423
         x3    -61.5201    48.3416   -1.2726     0.2213

Residual standard error: 300.5 on 16 degrees of freedom
Multiple R-Squared: 0.3464
F-statistic: 2.826 on 3 and 16 degrees of freedom, the p-value is 0.07178
```

The P values of the above summary tells us that (at 5% level):

(a) all $\beta_0, \beta_1, \beta_2$ and β_4 are significant (b) none of $\beta_0, \beta_1, \beta_2$ or β_4 is significant
(c) only β_1 and β_3 are significant (d) β_1 and β_3 are not significant
(e) β_0 and β_2 are not significant

THIS IS THE LAST QUESTION OF SECTION I

PLEASE TURN OVER FOR SECTION II (EXTENDED ANSWER SECTION)

.../4

Section II - Extended Answer Questions - 10 marks each

1. The merits of a new technique of a production line are under investigation to see if this technique gives a lower number of defective items in general than the old technique. Ten days of production are obtained by both techniques with the following results:

Day	1	2	3	4	5	6	7	8	9	10
New	172	165	206	184	174	142	190	169	161	200
Old	201	179	159	192	177	170	182	179	169	210

Using level $\alpha = 0.05$,

(a) Analyse the data using the sign test, mentioning any assumptions used. [HINTS: Write p_+ = probability that the new technique gives a lower defectives and set up the null hypothesis $H_0 : p_+ = 0.5$ and alternative $H_1 : p_+ < 0.5$. Notice that this is a one-sided test.]

(b) Analyse the data using the Wilcoxon signed rank test.

2. The waiting time, X between tremors recorded at a seismological station is believed to follow an exponetial distribution with parameter θ. That is, the probability density function of X is, $f(x) = \frac{1}{\theta}e^{\frac{-x}{\theta}}$, $x > 0$. A single observation, x is available and it is required to test the null hypothesis $H_0 : \theta = 10$ against the alternative $H_1 : \theta \neq 10$. The following test is proposed:

$$\text{Reject } H_0 \text{ if } x < 8 \text{ or } x > 12.$$

(a) What is the significance level of this test?

(b) Calculate the probabilities of type II errors when $\theta = 2$ and $\theta = 12$.

(c) Calculate the powers of the test when $\theta = 2$ and $\theta = 12$ (give your answers to three decimal places).

3. (a) The Kruskal-Wallis statistic for comparing k population means is given by $H = \frac{12}{n(n+1)}(\sum_{i=1}^{k} n_i(\bar{R}_i - \frac{n+1}{2})^2$, where R_i is the sum of ranks of the i^{th} sample consisting of n_i observartions, $\bar{R}_i = \frac{R_i}{n_i}$ is the average rank and $n = \sum_{i=1}^{k} n_i$. *Show* that $H = \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(n+1)$.

(b) An Environment Protection Agency (EPA) wants to determine whether temperature changes in the ocean's water caused by a nuclear power plant will have significant effect on the animal life in the region. Recently hatched specimens of a certain fish are randomly divided into four groups and are placed in seperate simulated ocean environments that are identical in every way except for water temperature. Six months later, the specimens are weighed. The results (in ounces) are given below under each temperature (in Centigrades):

Temperature			
6^0C	10^0C	14^0C	18^0C
16	15	14	13
18	16	23	18
22	25	28	20
	26		21

Using the Kruskal-Wallis test at $\alpha = 0.05$, test whether the data provide sufficient evidence to indicate that one (or more) of the temperatures tend(s) to produce larger weight increases than the other temperatures.

4. (a) A department store is considering building a new store at one of four different sites. One of the main factors in the decision is the annual income of the households in the four areas. In a preliminary study, a random sample of residents in each area is asked to state their annual income. The results and the totals for each column are given below:

Annual income			
Area 1	Area 2	Area 3	Area 4
42	18	27	30
26	12	51	29
29	20	19	17
23	25	22	31
18	48	32	27
	32	46	25
	27	30	
		35	
		32	
138	182	294	159

i. Given that $\sum\sum x_{ij} = 773$ and $\sum\sum x_{ij}^2 = 24493$, perform an analysis of variance to test the null hypothesis of no difference in mean incomes of four areas. Present your results as an ANOVA table. Use level $\alpha = 0.05$.

ii. The analysis of part (i) assumes the observations come from populations with the same variance σ^2. What other assumptions are necessary?

iii. Show that a point estimate of σ^2 is 92.8. Give a 90% confidence interval for the common variance, σ^2.

(b) A two factor experiment (factors are A and B) has been conducted to investigate the effect of interaction between the levels of each factor. Three different response levels for each factor have been used and three observations taken from each cell (that is, there is a total of 27 observations). The following part ANOVA table has been prepared and you are asked to conduct the test.

ANOVA Table

Source	df	SS	MS	F
Factor A	2	20.22	10.11	4.20
Factor B	*	8.22	*	1.71
Interaction $A \times B$	*	46.22	*	*
Error	*	*	*	
Total	26	118		

Fill the missing entries * of the above atble and test the null hypothesis of no interaction at 1% level of significance.

5. An insurance company wants to relate the amount of fire damage in major residential fires to the distance between the residence and the nearest fire station. The amount of damage y (in thousands of dollars) and the distance x (in km) between the fire and the nearest fire station are recorded for a sample of 15 recent fires. The following summary is available for the 15 pairs: $s_{xx} = 34.784$, $s_{yy} = 911.517$, and $s_{xy} = 171.114$.

(a) Estimate the Pearson's correlation coefficient, ρ between the two variables. Test the null hypothesis $H_0 : \rho = 0.9$.

(b) i. Write down a suitable linear regression model with all necessary assumptions.

ii. Estimate the regression line in (i) in order to predict the amount of damage.

iii. Find an estimate of the error variance, σ^2.

iv. Perform a statistical test to see whether the slope of the line is 5. Use $\alpha = 0.05$.

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Attachments

1. Multiple choice answer sheet

2. Statistical tables

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Multiple Choice Response Sheet

Mark your correct response clearly to each multiple choice question

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

PTO for Statistical Tables

TABLE I: (*continued*)

n	x	θ .05	.10	.15	.20	.25	.30	.35	.40	.45	.50
8	0	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039
	1	.2793	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0312
	2	.0515	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094
	3	.0054	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188
	4	.0004	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734
	5	.0000	.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188
	6	.0000	.0000	.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094
	7	.0000	.0000	.0000	.0001	.0004	.0012	.0033	.0079	.0164	.0312
	8	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0007	.0017	.0039
9	0	.6302	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.2985	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.0629	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0077	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4	.0006	.0074	.0283	.0061	.1168	.1715	.2194	.2508	.2600	.2461
	5	.0000	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6	.0000	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7	.0000	.0000	.0000	.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8	.0000	.0000	.0000	.0000	.0001	.0004	.0013	.0035	.0083	.0176
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0008	.0020
10	0	.5987	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.3151	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.0746	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0105	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4	.0010	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5	.0001	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6	.0000	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7	.0000	.0000	.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172
	8	.0000	.0000	.0000	.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0016	.0042	.0098
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0016
11	0	.5688	.3138	.1673	.0859	.0422	.0198	.0088	.0036	.0014	.0005
	1	.3293	.3835	.3248	.2362	.1549	.0932	.0518	.0266	.0125	.0054
	2	.0867	.2131	.2866	.2953	.2581	.1998	.1395	.0887	.0513	.0269
	3	.0137	.0710	.1517	.2215	.2581	.2568	.2254	.1774	.1259	.0806
	4	.0014	.0158	.0536	.1107	.1721	.2201	.2428	.2365	.2060	.1611
	5	.0001	.0025	.0132	.0388	.0803	.1321	.1830	.2207	.2360	.2256
	6	.0000	.0003	.0023	.0097	.0268	.0566	.0985	.1471	.1931	.2256
	7	.0000	.0000	.0003	.0017	.0064	.0173	.0379	.0701	.1128	.1611
	8	.0000	.0000	.0000	.0002	.0011	.0037	.0102	.0234	.0462	.0806
	9	.0000	.0000	.0000	.0000	.0001	.0005	.0018	.0052	.0126	.0269

TABLE III: Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4988
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Also, for $z = 4.0$, 5.0, and 6.0, the probabilities are 0.49997, 0.4999997, and 0.499999999.

TABLE IV: Values of $t_{\alpha,\nu}$†

ν	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$	ν
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
inf.	1.282	1.645	1.960	2.326	2.576	inf.

†Based on Richard A. Johnson and Dean W. Wichern, *Applied Multivariate Statistical Analysis*, 2nd ed., © 1988, Table 2, p. 592. By permission of Prentice Hall, Upper Saddle River, N.J.

TABLE V: Values of $\chi^2_{\alpha,\nu}$ †

ν	$\alpha = .995$	$\alpha = .99$	$\alpha = .975$	$\alpha = .95$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$	ν
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879	1
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597	2
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838	3
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860	4
5	.412	.554	.831	1.145	11.070	12.832	15.086	16.750	5
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548	6
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278	7
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955	8
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589	9
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188	10
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757	11
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300	12
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819	13
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319	14
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801	15
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267	16
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718	17
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156	18
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582	19
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997	20
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401	21
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796	22
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181	23
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.558	24
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928	25
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290	26
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645	27
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993	28
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336	29
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672	30

†Based on Table 8 of *Biometrika Tables for Statisticians*, Vol. 1, Cambridge University Press, 1954, by permission of the *Biometrika* trustees.

TABLE VI: Values of $f_{0.05,\nu_1,\nu_2}$ †

ν_1 = Degrees of freedom for numerator

ν_2 = Degrees of freedom for denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

†Reproduced from M. Merrington and C. M. Thompson, "Tables of percentage points of the inverted beta (*F*) distribution," *Biometrika*, Vol. 33 (1943), by permission of the *Biometrika* trustees.

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TABLE VI: (*continued*) Values of $f_{0.01,\nu_1,\nu_2}$

ν_1 = Degrees of freedom for numerator

ν_2 = Degrees of freedom for denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,235	6,261	6,287	6,313	6,339	6,366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	6.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

TABLE X: Critical Values for the Signed-Rank Test†

n	$T_{0.10}$	$T_{0.05}$	$T_{0.02}$	$T_{0.01}$
4				
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0
9	8	6	3	2
10	11	8	5	3
11	14	11	7	5
12	17	14	10	7
13	21	17	13	10
14	26	21	16	13
15	30	25	20	16
16	36	30	24	19
17	41	35	28	23
18	47	40	33	28
19	54	46	38	32
20	60	52	43	37
21	68	59	49	43
22	75	66	56	49
23	83	73	62	55
24	92	81	69	61
25	101	90	77	68

†From F. Wilcoxon and R. A. Wilcox, *Some Rapid Approximate Statistical Procedures*, American Cyanamid Company, Pearl River, N. Y., 1964. Reproduced with permission of American Cyanamid Company.

TABLE XI: Critical Values for the U Test†

Values of $U_{0.10}$

n_1 \ n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2				0	0	0	1	1	1	1	2	2	3	3
3		0	0	1	2	2	3	4	4	5	5	6	7	7
4		0	1	2	3	4	5	6	7	8	9	10	11	12
5	0	1	2	4	5	6	8	9	11	12	13	15	16	18
6	0	2	3	5	7	8	10	12	14	16	17	19	21	23
7	0	2	4	6	8	11	13	15	17	19	21	24	26	28
8	1	3	5	8	10	13	15	18	20	23	26	28	31	33
9	1	4	6	9	12	15	18	21	24	27	30	33	36	39
10	1	4	7	11	14	17	20	24	27	31	34	37	41	44
11	1	5	8	12	16	19	23	27	31	34	38	42	46	50
12	2	5	9	13	17	21	26	30	34	38	42	47	51	55
13	2	6	10	15	19	24	28	33	37	42	47	51	56	61
14	3	7	11	16	21	26	31	36	41	46	51	56	61	66
15	3	7	12	18	23	28	33	39	44	50	55	61	66	72

Values of $U_{0.05}$

n_1 \ n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2							0	0	0	0	1	1	1	1
3				0	1	1	2	2	3	3	4	4	5	5
4			0	1	2	3	4	4	5	6	7	8	9	10
5		0	1	2	3	5	6	7	8	9	11	12	13	14
6		1	2	3	5	6	8	10	11	13	14	16	17	19
7		1	3	5	6	8	10	12	14	16	18	20	22	24
8	0	2	4	6	8	10	13	15	17	19	22	24	26	29
9	0	2	4	7	10	12	15	17	20	23	26	28	31	34
10	0	3	5	8	11	14	17	20	23	26	29	30	36	39
11	0	3	6	9	13	16	19	23	26	30	33	37	40	44
12	1	4	7	11	14	18	22	26	29	33	37	41	45	49
13	1	4	8	12	16	20	24	28	30	37	41	45	50	54
14	1	5	9	13	17	22	26	31	36	40	45	50	55	59
15	1	5	10	14	19	24	29	34	39	44	49	54	59	64

†This table is based on D. Auble, "Extended Tables for the Mann–Whitney Statistics," *Bulletin of the Institute of Educational Research at Indiana University*, Vol. 1, 1953. By permission of the author.