8112

THE UNIVERSITY OF SYDNEY

FACULTIES OF ARTS AND SCIENCE

STAT 2012: Statistical Tests

Semester 2, 2007 Time allowed: Two hours

Answer ALL questions.
For each hypothesis test unless otherwise stated, you should state the two hypotheses, the test statistic, the test assumptions, the p-value and the conclusion. If an exact p-value cannot be found from the table, state the range of values from the table.

1. The marketing manager of a company studied the commission income (X in thousand dollars) of its salespersons. He conducted tests on the hypotheses:

$$H_0 : \ \mu = 15 \quad \text{against} \quad H_1 : \ \mu > 15$$

where μ denotes the mean of commission incomes from all salespersons. A random sample of 8 salespersons was drawn and their commission incomes were recorded as below:

11 12 15 16 18 20 24 40

The sample mean and sample variance are $\bar{x} = 19.5$ and $s^2 = 86.286$ respectively.

(a) (4 marks) State the rejection region for the sample mean if the test was conducted at the 5% significance level using a t-test. Will you reject the null hypothesis?

(b) (4 marks) The population variance of commission incomes is $\sigma^2 = 80$. Calculate the type II error of the z-test with a rejection region of $\bar{x} \geq 20$ if the true mean is $\mu = 18$.

(c) (5 marks) Calculate the p-value of the test if Wilcoxon signed rank test is used.

2. A researcher investigated if the ages of the audience who favor TV programs A and B differ. A random sample of 13 members of the audience was selected and their ages were recorded below:

Favor program A (x)	15	16	18	20	21	22	
Favor program B (y)	17	19	23	25	26	30	33

The following summary statistics are given:

$$\bar{x} = 18.6667, \quad \bar{y} = 24.7143, \quad s_x^2 = 7.8667, \quad s_y^2 = 32.2381.$$

(a) (5 marks) Test at the 5% significance level if the mean ages of the audience members differ between programs A and B using a two-sample t-test.

(b) (5 marks) Test at the 5% significance level if the mean ages of the audience members differ between programs A and B using a Wilcoxon rank sum test.

(c) (3 marks) Calculate a 95% confidence interval for the ratio of the variances of the ages of audience members who favor program B to those who favor program A.

3. In a randomized block design experiment, patients were grouped into five groups according to their weights. Patients in each group were then randomly assigned to three weight-reduction programs. The weight loss y_{ij} (in pounds) after the programs were recorded below:

Weight	**Programs**			
groups	A	B	C	Sum
I	11	12	8	31
II	21	8	9	38
III	15	16	13	44
IV	16	13	6	35
V	18	15	11	44
Sum	81	64	47	192

The following summary statistics are given:

$$\sum_{i=1}^{5}\sum_{j=1}^{3} y_{ij} = 192, \quad \sum_{i=1}^{5}\sum_{j=1}^{3} y_{ij}^2 = 2696.$$

(a) (6 marks) Complete the ANOVA table.

(b) (4 marks) Test at the 5% significance level if the means of weight loss differ across programs using an ANOVA test.

(c) (5 marks) Test at the 5% significance level if the means of weight loss differ across programs using the Friedman test.

4. A professor of economics studied the relationship between income (Y in thousand dollars) and education (X in years). A random sample of eight individuals was taken and the results are reported below:

Income Y	58	40	55	35	43	41	52	49
Education X	16	11	15	8	12	10	13	14

The following summary statistics are given:

$$\sum_{i=1}^{8} x_i = 99, \quad \sum_{i=1}^{8} y_i = 373, \quad \sum_{i=1}^{8} x_i^2 = 1275, \quad \sum_{i=1}^{8} y_i^2 = 17849, \quad \sum_{i=1}^{8} x_i y_i = 4761.$$

(a) (4 marks) Determine the least squares regression line with Y as the dependent variable.

(b) (6 marks) Test the significance of the regression model in (a) at the 5% significance level. (Hint: Test if the slope of the regression line is zero.)

(c) (5 marks) Determine a 95% prediction interval for the income level of a person who has 10 years of education.

5. The personnel manager of a consumer product company asked a random sample of employees how they felt about the work they were doing. The following table gives a breakdown of the level of job satisfaction by gender.

	Response		
Gender	Very satisfied	Fairly satisfied	Not satisfied
Male	70	41	9
Female	35	34	11

(a) (6 marks) Compute the expected frequencies for the six cells under the assumption of independence between the factors of gender and the level of job satisfaction.

(b) (3 marks) Test at the 5% significance level the assumption of independence between the factors of gender and the level of job satisfaction.

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