THE UNIVERSITY OF SYDNEY

SCHOOL OF MATHEMATICS AND STATISTICS

Semester 2	STAT2012 Statistical Tests - QUIZ (Sept. 16)	2015

SAMPLE QUIZ - Time allowed 40 minutes

Instructions: The quiz covers course material from Week 1 to Week 7 (inclusive). Mark one answer to each question in pen.

1. The confidence interval estimate for a population mean μ will become *wider* if which of the following conditions change while other conditions remain unchanged.

   (a) the sample size n increases
   (b) the level of significance α increases
   (c) the sample variance s^2 increases
   (d) the sample mean $\bar{x}$ increases
   (e) none of the above

2. Calculate the probability of a Type II error (β) for the following test of hypothesis:

$$H_0: \ \mu = 50 \quad \text{vs} \quad H_1: \ \mu > 50$$

   given that $\mu = 55$, $\sigma = 8$, $n = 16$ and the rejection region is $\bar{x} \geq 54$.
   (a) 0.0062 (b) 0.9938 (c) 0.0228 (d) 0.9772 (e) 0.3085

3. Wilcoxon rank sum test is used to test the following data if the first population location is greater than the second population location. State the test statistic and p-value of the test.

```
Sample 1:  33 38 26 21
Sample 2:  20 18 25 16 22
```

   (a) $W = 29$ and p-value=0.0095
   (b) $W = 27$ and p-value=0.0556
   (c) $W = 12$ and p-value=0.0190
   (d) $W = 30$ and p-value=0.0079
   (e) $W = 28$ and p-value=0.0317

4. In a hypothesis test for the population variance, the hypotheses are
   $H_0: \ \sigma^2 = 30$ vs $H_1: \ \sigma^2 < 30$.
   If the sample size is 20, the sample variance is s^2 and the test is being carried out at the 5% level of significance, the null hypothesis will be rejected if:

   (a) $\chi_0^2 = 19(30)/s^2 > 30.144$
   (b) $\chi_0^2 = 20s^2/30 < 10.851$
   (c) $\chi_0^2 = 20(30)/s^2 > 31.410$
   (d) $\chi_0^2 = 19s^2/30 < 10.117$

(e) $\chi_0^2 = 19s^2/30 < 30.144$

5. Which of the following statements is correct regarding the percentile points of the F distribution?

   (a) $F_{10,20,0.05} = 1/F_{10,20,0.95}$
   (b) $F_{10,20,0.05} = 1/F_{20,10,0.05}$
   (c) $F_{10,20,0.95} = F_{20,10,0.95}$
   (d) $F_{10,20,0.95} = 1/F_{20,10,0.05}$
   (e) $F_{10,20,0.05} = 1/F_{10,20,0.05}$

6. Random samples from two normal populations produced the following statistics:
   $n_x = 10 \quad \bar{x} = 200 \quad s_x^2 = 225$
   $n_y = 20 \quad \bar{y} = 210 \quad s_y^2 = 100$
   What are the test statistic and the p-value for the test if the two population means differ?

   (a) $t_0 = -2.18$ and $0.02 < 2\Pr(t_{28} \leq -2.18) < 0.05$
   (b) $t_0 = -5.63$ and $2\Pr(t_{28} \leq -5.63) < 0.002$
   (c) $t_0 = -0.18$ and $2\Pr(t_{28} \leq -0.18) > 0.5$
   (d) $t_0 = -2.22$ and $0.01 < \Pr(t_{28} \leq -2.22) < 0.025$
   (e) $t_0 = -5.23$ and $2\Pr(t_{30} \geq -5.25) < 0.002$

7. Refer to the previous question. What are the test statistic and the p-value for the test if the two population variances differ?

   (a) $f_0 = 1.5$ and $2\Pr(F_{9,19} \geq 1.5) > 0.2$
   (b) $f_0 = 2.25$ and $0.1 < 2\Pr(F_{9,19} \geq 2.25) < 0.2$
   (c) $f_0 = 0.67$ and $2\Pr(F_{19,9} \leq 0.67) > 0.1$
   (d) $f_0 = 0.44$ and $0.9 < \Pr(F_{9,19} \geq 0.44) < 0.95$
   (e) $f_0 = 2.25$ and $2\Pr(F_{19,9} \geq 2.25) > 0.2$

8. A matched pairs experiment yielded the following results.

```
Number of positive differences = 3
Number of zero differences = 2
Number of negative differences = 7
```

   We want to test if the mean for one population is less than the mean of the other population. What is the p-value of the test?

   (a) 0.1719 (b) 0.6230 (c) 0.3872 (d) 0.0730 (e) 0.0547

9. Calculate the Kruskal- Wallis test statistics for the data with summary information as below using the approximated formula under no ties condition.

```
sum ranks in group 1 =  884  n1 =20
sum ranks in group 2 =  756  n2 =28
sum ranks in group 3 = 1061  n3 =25
```

   (a) 12.371 (b) 4.812 (c) 5.463 (d) 10.167 (e) 16.230

10. One-way ANOVA is applied to independent samples taken from three normally distributed populations with equal variances. The following summary statistics were calculated:

$$\begin{array}{lll} n_1 = 10 & \overline{y}_1 = 40 & s_1 = 5 \\ n_2 = 10 & \overline{y}_2 = 48 & s_2 = 6 \\ n_3 = 10 & \overline{y}_3 = 50 & s_3 = 4 \end{array}$$

The between-treatments variation SST is

(a) 460 (b) 688 (c) 560 (d) 183 (e) 693

****

Question	Your choice
1	a b c d e
2	a b c d e
3	a b c d e
4	a b c d e
5	a b c d e
6	a b c d e
7	a b c d e
8	a b c d e
9	a b c d e
10	a b c d e

****

Name:

Tutorial class and time:

Room:

Score:

Comments

If your score is:

≤ 4 - Study very hard. Re-do all tutorial & problem sheets.
$5, 6$ - Good, but practice more problems.
$7, 8$ - Very good. Keep revising.
≥ 9 - Excellent. Maintain this level.

Solution

1. c
2. e $\Pr(\bar{x} < 54) = \Pr(z < \frac{54-55}{8/\sqrt{16}}) = \Pr(z < -0.5) = 1 - 0.6915 = 0.3085$
3. e Rank: 8 9 7 4   3 2 6 1 5 $W = 28$ and
   p-value $= \Pr(W \geq 28) = \Pr(W \leq n_1(N+1) - 28) = \Pr(W \leq 4(10) - 28) = \Pr(W \leq 12) = 0.0317$
   (table, $n = 4, 5,\ w = 12$)
4. d $\chi_0^2 = (n-1)s^2/\sigma_0^2 = 19s^2/30$ and rr is $\chi_0^2 \leq \chi^2_{19,0.05} = 10.117$, i.e. $19s^2/30 \leq 10.117$
5. d
6. a $s_p^2 = [(n_x - 1)s_x^2 + (n_y - 1)s_y^2]/(n_x + n_y - 2) = [(9)225 + (19)100]/(28) = 140.1786$
   $t_0 = \dfrac{\bar{x} - \bar{y}}{\sqrt{s_p^2(1/n_1 + 1/n_2)}} = \dfrac{200 - 210}{\sqrt{140.1786(1/10 + 1/20)}} = -2.18$, $0.02 <$ p-value $= 2\Pr(t_{28} < -2.18) < 0.05$
7. b $f_0 = s_x^2/s_y^2 = 225/100 = 2.25$ and $0.1 <$ p-value $= 2\Pr(F_{9,19} \geq 2.25) < 0.2$
8. a $X = \#(d_i > 0) = 3$, $X \sim \mathcal{B}(10, 0.5)$, p-value $= \Pr(X \leq 3) = 0.1719$
   (Table, $n = 10$, $p = 0.5$, $x = 3$).
9. d $k_0 = \dfrac{12}{N(N+1)} \sum_i n_i \bar{r}_i^2 - 3(N+1) = \dfrac{12}{73(74)}(\dfrac{884^2}{20} + \dfrac{756^2}{28} + \dfrac{1061^2}{25}) - 3(74) = 10.167$
10. c $\bar{y} = \frac{1}{3}(40 + 48 + 50) = \frac{138}{3} = 46$, $SST = \sum_i n_i \bar{y}_i^2 - \bar{y}^2 = 10(40^2 + 48^2 + 50^2) - 30(46^2) = 560.$