

Semester 2	Tutorial Week 13	2015
------------	------------------	------

Summary of week 12

- *Chi-square* test for homogeneity and independence for 2-way data:

If y_{ij} are $r \times c$ observed frequencies, the test statistic for testing the *homogeneity* of distributions for each level of factor 1 across levels of factor 2 (or vice versa) with the hypotheses:

$$H_0 : p_{ij} = p_{i'j}, \text{ for all } i, i' = 1, \dots, r; j = 1, \dots, c \quad \text{vs} \quad H_1 : \text{Not all equalities hold}$$

or for testing the *independence* between two factors with the hypotheses:

$$H_0 : p_{ij} = p_{i.}p_{.j}, \text{ for all } i = 1, \dots, r; j = 1, \dots, c \quad \text{vs} \quad H_1 : \text{Not all equalities hold}$$

is

$$\chi_0^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{\left(y_{ij} - \frac{y_{i.}y_{.j}}{n}\right)^2}{\frac{y_{i.}y_{.j}}{n}} = \sum_{i=1}^r \sum_{j=1}^c \frac{(y_{ij} - np_{i.}p_{.j})^2}{np_{i.}p_{.j}} \sim \chi_{(r-1)(c-1)}^2$$

Note that the $\chi_{(r-1)(c-1)}^2$ approximation is good only when the expected frequencies $E_{ij} = np_{i.}p_{.j} \geq 5$.

Tutorial Questions

1. The following data are believed to have come from a normal probability distribution.

14 16 18 20 | 21 21 21 21 22 23 23 23 24 24 24 25 25 26 26 26
26 26 26 26 | 27 27 29 29 30 30 31 32 32 32 33 | 35 36 36 40 46

The mean of this sample equals 26.80, and the standard deviation equals 6.378. Use the goodness-of-fit test at the 5% significance level to test this claim. Given $P(Z > 1) = 0.159$.

2. A study of the amount of violence viewed on television as it relates to the age of the viewer yields the results shown in the accompanying table for 81 people.

	Age		
Viewing	16-34	35-54	55 and Over
Low violence	8	12	21
High violence	18	15	7

Do the data indicate that viewing of violence is not independent of age of viewer?

3. The numbers of customers i who enter a bank in 130 5-minutes intervals are as follow:

i	0	1	2	3	4	5	6	7	Sum
y_i	7	18	38	29	16	12	8	2	130

Test if the data follow a Poisson distribution. Note that $P(X = x) = \lambda^x e^{-\lambda} / x!$.

Extra Practice Problems

1. 103 children attending a pre-school were classified by parents income group and by IQ (intelligence quotient).

		High IQ	Moderate/low IQ
Income group	A	14	18
	B	25	8
	C	23	15

Do these data suggest that the fractions of IQ differ in the three income groups?