THE UNIVERSITY OF SYDNEY
STAT2012 STATISTICAL TESTS

Semester 2	**Tutorial Week 2**	2015

You need normal and t distribution tables.

Summary of week 1

Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from a normal population. To test the null hypothesis $H_0 : \mu = \mu_0$, use the statistic:

- $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$, when the population sd σ is known,
- $t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$, when the population sd σ is unknown, where s is the sample sd and $t \sim t_{n-1}$.

Tutorial questions

1. **Multiple choice** $X_1, X_2, \ldots, X_{16}$ represents a random sample from a normal distribution with mean $\mu = 20$ and unknown standard deviation. Indicate which of the following distributions is a good approximation to the distribution of $\dfrac{\bar{X} - 20}{S/4}$ where $\bar{X}$ is the sample mean and S is the sample deviation:

   (a) $\mathcal{N}(20, s^2)$ (b) $\mathcal{N}(0, 1)$ (c) $\mathcal{N}(20, s^2/16)$ (d) t_{16} (e) t_{15}

2. **Multiple choice** Suppose that Sandy wishes to test the hypotheses $H_0 : \mu = 60$ against $H_1 : \mu > 60$ for a normal population with an unknown standard deviation. A random sample of size 30 from this population gave her a mean of 63 and a standard deviation of 9. The observed value, $t_0 = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$, of the test statistic under the null hypothesis is:

   (a) 2.03 (b) 0.20 (c) 1.83 (d) 0.18 (e) 2.83

3. **Multiple choice** In Q2, you tell Sandy to perform a

   (a) one-sided t test and calculate the P-value using $P(t_{29} > 2.03)$
   (b) two-sided t test and calculate the P-value using $P(t_{29} > 2.03)$
   (c) two-sided t test and calculate the P-value using $P(t_{29} \neq 1.83)$
   (d) one-sided t test and calculate the P-value using $P(t_{29} > 1.83)$
   (e) one-sided t test and calculate the P-value using $P(|\bar{X} - 63| > 2)$

4. **Multiple choice** Suppose that we wish to test the hypotheses $H_0 : \mu = 6$ against $H_1 : \mu > 6$. Based on 16 observations from a normal population with $\sigma^2 = 4$ it is found that the sample mean, $\bar{x} = 7$. Using the statistic $Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$, the observed z_0 under the null hypothesis is

   (a) 1.2 (b) 2.0 (c) 4.0 (d) 0.0 (e) none of these

5. **Multiple choice** Information in Q4 tells us that:

    (a) the p-value is 0.9772 and we have strong evidence against H_0.
    (b) the p-value is 0.9772 and the data are consistent with H_0.
    (c) the p-value is 0.0228 and we have strong evidence against H_0.
    (d) the p-value is greater than 0.0228 and the data are consistent with H_0.
    (e) the p-value is 0.0 and the data is consistent with H_0.

6. The amount of sales in a drug store last week are 12.9, 13.5, 13.8, 13.3, 13.9, 14.2 (in thousand dollars) with the sample mean $\overline{x} = 13.6$ and the sample standard deviation $s = 0.46$. Test, at 0.05 significance level, whether the sales volume in last week is the same as the usual volume of 13.2 thousands assuming that

    (a) the population standard deviation σ is unknown.
    (b) the population standard deviation is known to be $\sigma = 0.5$.

Extra Practice Problems

1. Q7.4.21 (p405 of Larsen and Marx)
   A manufacturer of pipe for laying undergroud electrical cables is concerned about the pipe's rate of corrosion and whether a special coating may retard that rate. As a way of measuring corrosion, the manufacturer examines a short length of pipe and records the depth of the maximum pit. The manufacturer's tests have shown that in a year's time in the particular kind of soil the manufacturer must deal with, the average depth of the maximum pit in a foot of pipe is 0.0042 inches. To see whether that average can be reduced, 10 pipes are coated with a new plastic and buried in the same soil. After one year, the following maximum pit depths are recorded (in inches): 0.0039, 0.0041, 0.0038, 0.0044, 0.0040, 0.0036, 0.0034, 0.0036, 0.0046, and 0.0036. Given that the sample standard deviation for these 10 measurements is 0.00383 inches, can it be concluded at the $\alpha = 0.05$ level of significance that the plastic coating is benefical?

2. Q6.2.1 (p359 of Larsen and Marx)
   State the decision rule that would be used to test the following hypotheses. Evaluate the appropriate test statistic and state your conclusion.

    (a) $H_0 : \mu = 120$ verse $H_1 : \mu < 120$; $\bar{y} = 114.2,\ n = 25,\ \sigma = 18,\ \alpha = 0.08$.
    (b) $H_0 : \mu = 42.9$ verse $H_1 : \mu \neq 42.9$; $\bar{y} = 45.1,\ n = 16,\ \sigma = 3.2,\ \alpha = 0.01$.
    (c) $H_0 : \mu = 14.2$ verse $H_1 : \mu > 14.2$; $\bar{y} = 15.8,\ n = 9,\ \sigma = 4.1,\ \alpha = 0.13$.

3. Q6.2.7 (p360 of Larsen and Marx)
   The following are 30 blood alcohol determinations made by Analyzer GTE-10, a three-year-old unit that may be in need of recalibration. All 30 measurements were made using a test sample on which a properly adjusted machine would give a reading of 12.6%.

12.3	12.7	13.6	12.7	12.9	12.6
12.6	13.1	12.6	13.1	12.7	12.5
13.2	12.8	12.4	12.6	12.4	12.4
13.1	12.9	13.3	12.6	12.6	12.7
13.1	12.4	12.4	13.1	12.4	12.9

(a) If μ denote the true average reading that Analyzer GTE-10 would make on a person whose blood alcohol concentration is 12.6%, test

$$H_0 : \mu = 12.6 \quad \text{versus} \quad H_1 : \mu \neq 12.6$$

at the $\alpha = 0.05$ level of significance. Assume that $\sigma = 0.4$. Would you recommand that the machine be readjusted?

(b) What statistical assumptions are implicit in the hypothesis test done in Part (a)? Is there any reason to suspect that those assumptions may not be satisfied?

4. Suppose that Candi wishes to test the hypotheses $H_0 : \mu = 50$ against $H_1 : \mu \neq 50$ for a normal population with an unknown standard deviation. A random sample of size 25 from this population gave her a mean of 48 and a standard deviation of 4. Compute the P-value and draw your conclusion. Explain your decision to Candi.