THE UNIVERSITY OF SYDNEY
STAT2012 STATISTICAL TESTS

Semester 2	**Tutorial Week 3**	2015

You need normal and t distribution tables.

Summary of week 2

Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from a normal population.

- To test the null hypothesis $H_0 : \mu = \mu_0$, the decision rules using rejection region for the t-test and z test are:

$$\begin{array}{llll} \{\bar{x} : \bar{x} \geq k_0 = \mu_0 + t_{n-1}(\alpha)s/\sqrt{n}\}, & \text{or} & \{\bar{x} : \bar{x} \geq k_0 = \mu_0 + z(\alpha)\sigma/\sqrt{n}\} & \text{for } H_1: \ \mu > \mu_0; \\ \{\bar{x} : \bar{x} \leq k_0 = \mu_0 - t_{n-1}(\alpha)s/\sqrt{n}\}, & \text{or} & \{\bar{x} : \bar{x} \leq k_0 = \mu_0 - z(\alpha)\sigma/\sqrt{n}\} & \text{for } H_1: \ \mu < \mu_0; \\ \{\bar{x} : \bar{x} \leq k_0 = \mu_0 - t_{n-1}(\frac{\alpha}{2})s/\sqrt{n} \text{ or}, & \text{or} & \{\bar{x} : \bar{x} \leq k_0 = \mu_0 - z(\frac{\alpha}{2})\sigma/\sqrt{n} \text{ or} & \text{for } H_1: \ \mu \neq \mu_0; \\ \quad \bar{x} \geq k_0 = \mu_0 + t_{n-1}(\frac{\alpha}{2})s/\sqrt{n}\} & & \quad \bar{x} \geq k_0 = \mu_0 + z(\frac{\alpha}{2})\sigma/\sqrt{n}\} & \end{array}$$

- The type I error for the z-test is $\alpha = \Pr(\text{reject } H_0 |\ H_0 \text{ is true.}) = \Pr(z_0 \geq z(\alpha) |\ \mu = \mu_0)$

- The type II error for the z-test is

$$\begin{aligned} \beta_\alpha(\mu_1) &= \Pr(\text{accept } H_0 |\ H_0 \text{ is false and } \mu = \mu_1) \\ &= \Pr(z_0 < z(\alpha) |\ \mu = \mu_1 \geq \mu_0) \end{aligned}$$

- The Power curve for the z-test is

$$\text{Power}(\mu_1) = \Pr(\text{reject } H_0 |\ H_0 \text{ is false and } \mu = \mu_1) = \Pr(z_0 \geq z(\alpha) |\ \mu = \mu_1 \geq \mu_0)$$

Tutorial questions

1. The amount of sales in a drug store last week are 12.9, 13.5, 13.8, 13.3, 13.9, 14.2 (in thousand dollars) with the sample mean $\overline{x} = 13.6$ and the sample standard deviation $s = 0.46$. We want to test whether the sales volume in last week is the same as the usual volume of 13.2 thousands *assuming that the population standard deviation σ is known to be 0.5*. Take the level of significance to be $\alpha = 0.05$.

   (a) State the null and alternative hypotheses.

   (b) Find the critical value of the test. Hence find the rejection region for the sample mean $\bar{x}$. Find also the 95% confidence interval for μ.

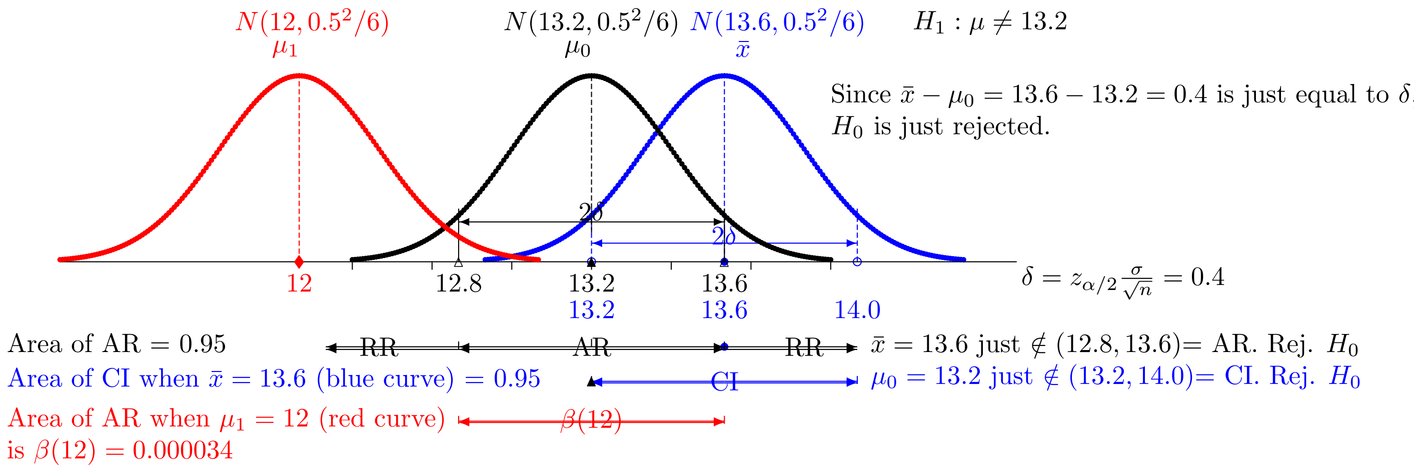
   (c) By comparing the sample mean with the rejection region, state your decision on whether you will reject the null hypothesis.

   (d) Recall that the p-value is 0.05. Do you get the same decision based on the p-value?

(e) With the rejection region in (b), calculate the type I α and type II error $\beta(12)$ (that is $\mu_1 = 12$ under $H_1 : \mu \neq 13.2$).

(f) Find the Power of the test at $\mu = 12$.

The different values can be illustrated in the following graph:



Extra Practice Problems

1. For the test of hypothesis

$$H_0 : \mu = 950 \quad \text{versus} \quad H_1 : \mu \neq 950,$$

calculate the following values given that $\mu = 1000$, $\alpha = 0.10$, $\sigma = 200$ and $n = 25$.

(a) Find the rejection region. Hence calculate the probability of Type II error β.

(b) Recalculate β if $n = 40$. What is the effect of increasing the sample size on the value of β?

(c) Recalculate β if α is lower from 0.1 to 0.05. What is the effect of increasing the significance level on the value of β?

(d) Using (a), calculate the power of the test and intepret your answer.

2. A random sample of 250 households in a large city revealed that the mean number of television per household was 2.76. From the previous analyses we know that the population standard deviation is 1.8. Can we conclude at the 5% significance level that the true number of televisions per household is at least 2.5? Compute the Type II error when the true mean is 3.

3. Suppose that Candi wishes to test the hypotheses $H_0 : \mu = 50$ against $H_1 : \mu < 50$ for a normal population with an unknown standard deviation. A random sample of size 25 from this population gave her a mean of 48 and a standard deviation of 4. Compute the rejection region on the sample mean at the $\alpha = 0.05$ level of significance and draw your conclusion. Explain your decision to Candi.