Semester 2	**Tutorial Week 4**	2015

Summary of week 3

- **Binomial test:** If X is the count of successes from n independent and identical trials with success probability p, the test statistic for the hypotheses

$$H_0 : p = p_0 \quad \text{vs} \quad H_1 : p > p_0, \quad \text{or} \quad p < p_0, \quad \text{or} \quad p \neq p_0$$

  is $\quad X \sim \mathcal{B}(n, p_0)$ under H_0.
  When n is large, $X \sim \mathcal{N}(np_0, np_0(1 - p_0))$ approximately.

- **Sign test:** If $X_1, \ldots, X_n$ are follow a continuous symmetric distribution, the test statistic for the hypotheses

$$H_0 : \mu = \mu_0 \quad \text{vs} \quad H_1 : \mu > \mu_0, \quad \text{or} \quad \mu < \mu_0, \quad \text{or} \quad \mu \neq \mu_0$$

  or equivalently

$$H_0 : p_+ = 0.5 \quad \text{vs} \quad H_1 : p_+ > 0.5, \quad \text{or} \quad p_+ < 0.5, \quad \text{or} \quad p_+ \neq 0.5$$

  is $\quad X = \#(d_i = x_i - \mu_0 > 0) \sim \mathcal{B}(n, 0.5)$ under H_0 where
  $p_+ = \Pr(X_i > \mu_0)$ and $\#(d_i = x_i - \mu_0 > 0)$ is the number of X_i greater than μ_0.

  When n is large, $X \sim \mathcal{N}(n/2, n/4)$ approximately under H_0.

- **Wilcoxon sign-rank test:** If $X_1, \ldots, X_n$ are follow a continuous symmetric distribution, the test statistic for the hypotheses

$$H_0 : \mu = \mu_0 \quad \text{vs} \quad H_1 : \mu > \mu_0, \quad \text{or} \quad \mu < \mu_0, \quad \text{or} \quad \mu \neq \mu_0$$

  is $\quad W^+ = \sum\limits_{i \,:\, x_i - \mu_0 > 0} r_i \quad \text{or} \quad w = \min(w^+, w^-)$ where $w^- = \sum\limits_{i \,:\, x_i - \mu_0 < 0} r_i$ and $r_i = \text{Rank}(|x_i - \mu_0|)$.
  When n is large or with 0 or with ties, $W^+ \sim \mathcal{N}(E(W^+), Var(W^+))$ approximately under H_0 and

$$
\begin{aligned}
E(W^+) &= \frac{1}{2} \sum_{i \,:\, x_i - \mu_0 \neq 0} r_i = \frac{1}{4} n(n + 1), \\
Var(W^+) &= \frac{1}{4} \sum_{i \,:\, x_i - \mu_0 \neq 0} r_i^2 \overset{\text{no ties}}{=} \frac{1}{24} n(n + 1)(2n + 1).
\end{aligned}
$$

Tutorial Questions

1. A biochemical substance is believed to have an inhibitive effect on muscular growth. Ten laboratory rats of similar types are selected. For each rat one hind leg was regularly injected with the biochemical substance. The corresponding muscle on the other hind leg was regularly injected with a harmless placebo. At the end of 6 months the weights of the muscles were measured (in gms) and recorded as follows.

Rat	1	2	3	4	5	6	7	8	9	10
Bioch. x_i	1.7	2.0	1.7	1.5	1.6	2.4	2.3	2.4	2.4	2.6
Placebo y_i	2.1	1.8	2.2	2.2	1.5	2.9	2.9	2.4	2.6	2.5

Suppose that the distribution is symmetric but the normality assumption is doubtful. Test whether this experiment provides evidence of a significant inhibitive effect using the *sign* test.

2. In a study of drug abuse in a suburban area, invertigators found that the median IQ of arrested abusers who were 16 years of age or older was 107. The following table show the IQs of a random sample of 15 persons from another suburban area.

Subject	1	2	3	4	5	6	7	8	9
IQ	100	90	135	108	107	119	127	109	105

Assuming that the population is symmetric, could the researchers concluded, at the 0.05 level of significance, that the mean IQ of arrested abusers who are 16 or older from the population of interest is higher than 107 using the Wilcoxon Signed Rank test?

3. Show that when $n = 3$, the probabilities $\Pr(W^+ \leq w^+)$ where $w^+ = 0, 1, 2, 3$ are 0.125, 0.250 and 0.375 respectively as given in the Wilcoxon Signed Rank Distribution (WSRD) table.

Extra Practice Problems

1. The proportion of defective items produced by a factory is 0.1. As part of the quality control procedure, a random sample of 12 items is inspected daily.

   (a) State the distribution of X, the number of defective items in a random sample of size 12.

   (b) Find the probability that there are fewer than two defective items on a particular day.

   (c) A new manager introduces work practices which are expected to reduce the proportion of defective items produced. After a settling-in period, he asks for a random sample of 200 items to be inspected for defects. Test the effectiveness of the new work practices if it is found that there are only 11 defective items in the sample.

   [Hints: Let p be the proportion of defective items produced after the new work practices are introduced. Set up appropriate null and alternative hypotheses concerning p.]

2. Suppose that the probability of success at each repetition of an experiment is 0.6. Perform a one-sided test of $H_0 : p = 0.6$ *vs.* $H_1 : p > 0.6$ if 40 successes are observed in 50 trials. Comment on your answers.

3. Ten secretaries were selected at random from among the secretaries of a large university. The typing speed (number of words per minute) was recorded for each secretary on two different brands of computer keyboards. The following results were obtained.

| | **Computer Keyboard** | |
Secretary	Brand A	Brand B
Amy	74	72
Betty	80	86
Carol	68	72
Donna	74	70
Ellen	86	85
Faith	75	73
Gwen	78	72
Heather	69	65
Ingrid	76	79
Katie	77	75

Assume that the typing speeds are not normally distributed. Perform the *sign* test to determine if these data provide enough evidence at the 5% significance level to infer that the brands differ with respect to typing speed.

4. Refer to Question 3. Perform the *Wilcoxon signed rank* test at the 5% level of significance.