THE UNIVERSITY OF SYDNEY

STAT2012 STATISTICAL TESTS

Semester 2	**Tutorial Week 5**	2015

Summary of week 4

- Two-sample t-test:

  If $X_1, X_2, \cdots, X_{n_x}$ and $Y_1, Y_2, \cdots, Y_{n_y}$ are two independent samples from $\mathcal{N}(\mu_x, \sigma^2)$ and $\mathcal{N}(\mu_y, \sigma^2)$ respectively, the test statistic is

  $$t_0 = \frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{s_p\sqrt{1/n_x + 1/n_y}} \sim t_{n_x+n_y-2},$$

  where $s_p^2 = \frac{(n_x-1)s_x^2+(n_y-1)s_y^2}{n_x+n_y-2}$ is the pooled variance estimate based on s_x^2 and s_y^2. If both n_x and n_y is large, the assumption of equal variance can be dropped. The test statistic is

  $$t_0 = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} \sim t_\nu$$

  where $\nu = \frac{\left[s_x^2/n_x + s_y^2/n_y\right]^2}{\frac{(s_x^2/n_x)^2}{n_x-1} + \frac{(s_y^2/n_y)^2}{n_y-1}}$. When both n_x and n_y are large, t_v is approx. $\mathcal{N}(0,1)$.

- Two sample z-test:

  If $X_1, X_2, \cdots, X_{n_x}$ and $Y_1, Y_2, \cdots, Y_{n_y}$ are two independent samples from $\mathcal{N}(\mu_x, \sigma_x^2)$ and $\mathcal{N}(\mu_y, \sigma_y^2)$ respectively where both σ_x^2 and σ_y^2 are known, the test statistic is

  $$z_0 = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim \mathcal{N}(0,1)$$

- Wilcoxon rank-sum test:

  If $X_1, X_2, \cdots, X_{n_x}$ and $Y_1, Y_2, \cdots, Y_{n_y}$ are two independent samples from the same kind of distribution differ by a shift, the test statistic is

  $$w = \sum_{i=1}^{n_x} \text{Rank}_i$$

  where Rank_i is the rank of the ith observation in the pooled sample. Without ties or n_x & n_y are both small, the p-value can be found from table. Otherwise, the test statistic is

  $$\begin{aligned} z_0 &= \frac{w - E(W)}{\sqrt{Var(W)}} \sim \mathcal{N}(0,1), \\ \text{where} \quad E(W) &= \frac{n_x(N+1)}{2} \quad \text{and} \\ Var(W) &= \frac{n_x n_y}{N(N-1)}\left(\sum_{j=1}^{N} r_j^2 - \frac{N(N+1)^2}{4}\right). \end{aligned}$$

Tutorial Questions

1. Two methods labelled A and B are used to measure the latent heat of fusion of ice.

A	79.98	80.04	80.02	80.04	80.03	80.03	80.04	79.97
	80.05	80.03	80.02	80.00	80.02			
B	80.02	79.94	79.98	79.97	79.97	80.03	79.95	79.97

Test the claim that method A gives higher measurements using the two-sample t-test. It is known that the sample means and sample s.d. are $\bar{x} = 80.02$, $s_x = 0.02397$, $\bar{y} = 79.98$ and $s_y = 0.03137$ respectively.

2. The pig's weights under diet X or Y are given in the following table. Test if there is a difference in weight using the Wilcoxon rank sum test.

X	12	16	16	12	10
Y	30	12	24	32	24

You may calculate the exact p-value by deriving the exact distribution.

3. Show that when $m = 3$ and $n = 3$, the probabilities $\Pr(W \leq w)$ where $w = 6, \ldots, 11$ are 0.05, 0.10, $\ldots$, 0.65 respectively as given in the Wilcoxon Rank Sum Distribution (WSRD) table.

4. Prove that when W is approximated by a normal distribution, the variance of W is given by

$$Var(W) = \frac{n_x n_y}{N(N-1)} \left(\sum_{i=1}^{N} r_i^2 - \frac{N(N+1)^2}{4} \right).$$

Extra Practice Problems

1. In a clinical trial, a new drug is tested if it will lower the cholesterol levels of patients. The following information is obtained from the treatment and control groups.

	Treatment group	Control group
Sample size	$n_x = 7$	$n_y = 8$
Sample mean	$\overline{x} = 6.34286$	$\overline{y} = 8.0625$
Sample variance	$s_x^2 = 1.40102^2$	$s_y^2 = 1.10446^2$

Test if the cholesterol levels of patients in treatment group is lower than those in control group using the two sample t-test.

2. Repeat the test in Question 1 without the assumption of equal variance.

3. From a group of nine rats available for a study of transfer of learning, five were selected at random and were trained to imitate leader rats in a maze. They were then placed together with four untrained control rats in a situation where imitation of the leaders enable them

to avoid receiving an electric shock. The results (the number of trials required to obtain ten correct responses in ten consecutive trials) were as follow:

Trained rats	78	64	75	45	82
Controls	110	70	53	51	

Test if there is a difference in the number of trials required between the trained rats and the controls using the Wilcoxon rank sum test.