THE UNIVERSITY OF SYDNEY

STAT2012 STATISTICAL TESTS

Semester 2	**Tutorial Week 6**	2015

Summary of week 5

- *Chi-square* test: Suppose that $X_1, X_2, \cdots, X_n$ is a random sample of size n from $N(\mu, \sigma^2)$.

  To test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \ \sigma^2 \neq \sigma_0^2$ (two-sided) or $H_1 : \ \sigma^2 > \sigma_0^2$ or $H_1 : \ \sigma^2 < \sigma_0^2$ (one-sided), the test statistic is

  $$\chi_0^2 = (n-1)s^2/\sigma_0^2 \sim \chi_{n-1}$$

  under H_0.

- *F-test:* Suppose that $X_1, X_2, \cdots, X_{n_x}$ and $Y_1, Y_2, \cdots, Y_{n_y}$ are two independent random samples from $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$ respectively.

  To test $H_0 : \sigma_x^2 = \sigma_y^2$ against $H_1 : \sigma_x^2 \neq \sigma_y^2$ (two-sided) or $H_1 : \sigma_x^2 > \sigma_y^2$ (one-sided), the test statistic ($s_x^2 > s_y^2$) is

  $$f_0 = \frac{s_x^2}{s_y^2} \sim F_{n_x-1, n_y-1}$$

  under H_0.

- ANOVA test: Suppose that $Y_{i1}, Y_{i2}, \cdots, Y_{in_i}$ are independent random samples from $N(\mu_i, \sigma^2)$.

  To test $H_0 : \ \mu_1 = \mu_2 = \cdots = \mu_g$ against H_1 : Not all the μ_i's are equal, the test statistic is

  $$f_0 = \frac{MST}{MSR} = \frac{\big[\sum_{i=1}^{g} n_i(\bar{y}_i - \bar{y})^2\big]/(g-1)}{\big[\sum_{i=1}^{g}(n_i-1)s_i^2\big]/(n-g)} \sim F_{g-1, n-g}$$

  under H_0.

Tutorial Questions

1. The closing prices of two common stocks were recorded for a period of 16 days. The means and variances were

   $$\bar{y}_1 = 40.33 \qquad \bar{y}_2 = 42.54$$
   $$s_1^2 = 1.54 \qquad s_2^2 = 2.96$$

   (a) Test if the variance of the closing prices of the first stock is greater than 1.

   (b) Test if the variances of the closing prices of the two stocks are the same.

   (c) Find a 95% confidence interval for the ratio of variances $\frac{\sigma_y^2}{\sigma_x^2}$.

2. An experiment was conducted to examine the effect of age on heartbeat rate when a subject is subjected to a specific amount of exercise. Male subjects were selected at random from four age groups. Each subject walked a treadmill at a fixed grade for 12 minutes and the increase in heartbeats per minute was recorded.

	Age range			
	10–19	20–39	40–59	60–69
	29	24	37	28
	33	27	25	29
	26	33	22	34
	27	31	33	36
	35	21	28	21
	33	28	26	20
	29	24	30	25
	36	34	34	24
	22	21	27	33
		32	33	32
Totals	270	275	295	282

$$CM = n\bar{y}^2 = 32279.08, \qquad \sum\sum x_{ij}^2 = 33180.$$

Test for differences in the mean increase among the four groups (construct the ANOVA table and draw your conclusion) and also test for the difference between the age group 10–19 and 20–39.

Extra Practice Problems

1. Construct a 95% confidence interval for the test of variance in question 1(a).

2. The irradiation of food to destroy bacteria is a growing phenomenon. In order to determine which one of two methods of irradiation is best, a scientist took a random sample of 52 one-pound packages of minced meat and subjected 31 of them to irradiation method 1 and the remaining 21 to irradiation method 2. The bacteria count was measured and the following statistics were computed.

Method 1	Method 2
$\bar{x} = 86$	$\bar{y} = 98$
$s_x^2 = 324$	$s_y^2 = 841$
$n_x = 31$	$n_y = 21$

The scientist noted that the data are normally distributed.

   (a) Determine whether these data are sufficient to infer at the 5% significance level that the two population variances differ.

   (b) Estimate with 95% confidence the ratio of the variances of the number of re-recordings of the two types of tape, and briefly describe what the interval estimate tells you.

3. The strength of a weld depends to some extent on the metal alloy used in the welding process. A scientist working in the research laboratory of a major automobile manufacturer has developed three new alloys. In order to test their strengths each alloy is used in several welds. The strength of the welds are then measured with the results shown below. Can the scientist conclude at the 5% significance level that differences exists among the strengths of the welds with the different alloys?

Strength of Welds

Alloy 1	Alloy 2	Alloy 3
15	17	25
23	21	27
16	19	24
29	25	31
	28	23
	19	