THE UNIVERSITY OF SYDNEY

STAT2012 STATISTICAL TESTS

Semester 2	**Tutorial Week 8**	2015

Summary of week 7

- *Two-way ANOVA test:* (Randomized block design)

  Suppose that Y_{ij} are independent random samples from block i and treatment j such that $Y_{ij} \sim \mathcal{N}(\mu_{ij}, \sigma^2)$ where $\mu_{ij} = \mu + \alpha_i + \beta_j$ and $\sum_{i=1}^{r} \alpha_i = \sum_{j=1}^{c} \beta_j = 0$.

  To test the hypotheses on treatment and block effects,

  $$\begin{array}{ll} H_0: \ \beta_1 = \cdots = \beta_c \quad \text{vs} \quad H_1: \ \text{at least one } \beta_j \text{ does not equal}, & \text{and/or} \\ H_0: \ \alpha_1 = \cdots = \alpha_r \quad \text{vs} \quad H_1: \ \text{at least one } \alpha_i \text{ does not equal}, & \end{array}$$

  the test statistics are respectively

  $$\begin{aligned} f_{t0} &= \frac{SST/(c-1)}{SSR/(r-1)(c-1)} \sim F_{c-1,(r-1)(c-1)}, \\ f_{b0} &= \frac{SSB/(r-1)}{SSR/(r-1)(c-1)} \sim F_{r-1,(r-1)(c-1)} \end{aligned}$$

  under H_0.

- *Friedman test:* (Randomized block design)

  Suppose that Y_{ij} are independent random samples from block i and treatment j.

  To test the hypotheses on treatment effects,

  $$H_0: \ \beta_1 = \cdots = \beta_c \quad \text{vs} \quad H_1: \ \text{at least one } \beta_j \text{ does not equal},$$

  the test statistic is

  $$q_0 = \frac{SST}{MST_0'} = \frac{r\sum_{j=1}^{c} \bar{r}_{\cdot j}^2 - rc(\bar{r})^2}{\left[\sum_{i=1}^{r}\sum_{j=1}^{c} r_{ij}^2 - rc(\bar{r})^2\right] / [r(c-1)]} \overset{\text{no ties}}{=} \frac{12r}{c(c+1)} \sum_{j=1}^{c} (\bar{r}_{\cdot j})^2 - 3r(c+1) \sim \chi_{c-1}$$

  under H_0 where r_{ij} is the rank of y_{ij} across block i.

Tutorial Questions

1. In a randomised blocked agricultural experiment to investigate four varieties of barley, the following yields (weights per plot in kilos minus 30) were obtained:

	Variety			
Block	A	B	C	D
I	3.5	8.2	1.8	3.2
II	4.1	8.0	0.5	5.2
III	5.4	10.6	3.4	5.7
IV	6.7	10.3	2.6	7.1
V	7.7	13.7	6.2	8.9

$$\sum_{i=1}^{r}\sum_{j=1}^{c} y_{ij} = 122.8, \quad \sum_{i=1}^{r}\sum_{j=1}^{c} y_{ij}^2 = 959.42.$$

Construct a two-way analysis of variance table of the above data and answer the following questions:

   (a) Is there evidence of differences in the yields among the four varieties of barley in terms of p-value?

   (b) Is there evidence of differences in the yields among the five blocks in terms of p-value?

   (c) Is there evidence of differences in the yields among the four varieties of barley if the block effect is dropped from the model?

2. The following data are measurements of the smoothness of five types of paper obtained in four different laboratories. Carry out a Friedman test of the null hypothesis of no systematic difference amongst the laboratories.

	Laboratory			
Type	A	B	C	D
I	38.7	39.2	34.0	34.0
II	41.5	39.3	35.0	34.8
III	43.8	39.7	39.0	34.8
IV	44.5	41.8	40.0	35.4
V	45.5	41.8	43.0	37.2

3. In the case of no ties,

   (a) Show that the Friedman test statistic is

$$q_0 = \frac{12r}{c(c+1)}\sum_{j=1}^{c}(\bar{r}_{\cdot j})^2 - 3r(c+1). \quad (1)$$

   Hint: note that $\bar{r} = (1+c)/2$ and in the case of no ties, $\sum_{j=1}^{c} r_{ij}^2 = \sum_{k=1}^{c} k^2 = \frac{1}{6}c(c+1)(2c+1)$.

   (b) Show that when $c = 2$, $q_0 = z_0^2$ where z_0 is the standardized test statistic for the *sign* test.

   (c) Show that when $c = 2$, $f_{t0} = t_0^2$ with df 1 and $n-1$ where t_0 is the test statistic for the *match pair* t-test and the number of blocks r is the number of pairs n.

Extra Practice Problems

1. In recent years the irradiation of food to reduce bacteria and preserve the food longer has become more common. A company that performs this service has developed four different methods of irradiation food. To determine which is best, it conducts an experiment where different foods are irradiated and the bacteria count is measured. As part of the experiment the following foods are irradiated: beef, chicken, turkey, eggs and milk. The results are shown below.

	Bacteria count			
	Method 1	Method 2	Method 3	Method 4
Beef	47	53	36	68
Chicken	53	61	48	75
Turkey	68	85	55	45
Eggs	25	24	20	27
Milk	44	48	38	46

   (a) Test if there are differences in the bacteria count among the four irradiation methods.

   (b) Test if there are differences in the bacteria count among the five types of food.

   (c) Repeat (a) using the Friedman test.