THE UNIVERSITY OF SYDNEY

STAT2012 STATISTICAL TESTS

Semester 2	**Tutorial Week 9**	2015

Summary of week 8

- *Two-way ANOVA test for 2-way data with replicates:* (Factorial design)

  Suppose that Y_{ijk} are independent random samples from block i and treatment j such that $Y_{ijk} \sim \mathcal{N}(\mu_{ij}, \sigma^2)$ where $\mu_{ij} = \mu + \alpha_i + \beta_j + \delta_{ij}$ and $\sum_{i=1}^{r} \alpha_i = \sum_{j=1}^{c} \beta_j = \sum_{i=1}^{r} \delta_{ij} = \sum_{j=1}^{c} \delta_{ij} = 0$.

  To test the hypotheses on treatment, block and interaction effects respectively as

$$\begin{array}{lcl} H_0: \ \beta_1 = \beta_2 = ... = \beta_c = 0 & \text{vs} & H_1: \text{ Not all } \beta_j \text{ are the same,} \\ H_0: \ \alpha_1 = \alpha_2 = ... = \alpha_r = 0 & \text{vs} & H_1: \text{ Not all } \alpha_i \text{ are the same,} \\ H_0: \ \delta_{ij} = 0, \ i = 1, ..., r; \ j = 1, ..., c & \text{vs} & H_1: \text{ Not all } \delta_{ij} \text{ are the same,} \end{array}$$

  the test statistics are respectively

$$\begin{aligned} f_{t0} &= \frac{SST/(c-1)}{SSR/[rc(m-1)]} \sim F_{c-1,rc(m-1)}, \\ f_{b0} &= \frac{SSB/(r-1)}{SSR/[rc(m-1)]} \sim F_{r-1,rc(m-1)}, \\ f_{i0} &= \frac{SSI/[(r-1)(c-1)]}{SSR/[rc(m-1)]} \sim F_{(r-1)(c-1),rc(m-1)} \end{aligned}$$

  under H_0 where

$$\begin{aligned} SST_o &= \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{m} {y_{ijk}}^2 - rcm\,\bar{y}^2, \\ SSB &= cm \sum_{i=1}^{r} y_{i..}^2 - rcm\,\bar{y}^2, \\ SST &= rm \sum_{j=1}^{c} y_{.j.}^2 - rcm\,\bar{y}^2, \\ SSR &= (m-1)\sum_{i=1}^{r}\sum_{j=1}^{c} s_{ij.}^2, \\ SSI &= SST_o - SSB - SST - SSR. \end{aligned}$$

Tutorial Questions

1. An experiment was designed to assess the effect of certain injection to release headache. Four injections were applied either once a week or once every two days. Additionally, four drug mixtures were used to test the effectiveness of different injections. Five headache patients were randomly selected for each combination of schedule and drug mixture. An index from 0 to 100 to measure the frequency, duration and severity of headache was measured prior to treatment and after follow-ups. Improvement in headache index was recorded for each patient as below:

Improvement in Headache Index

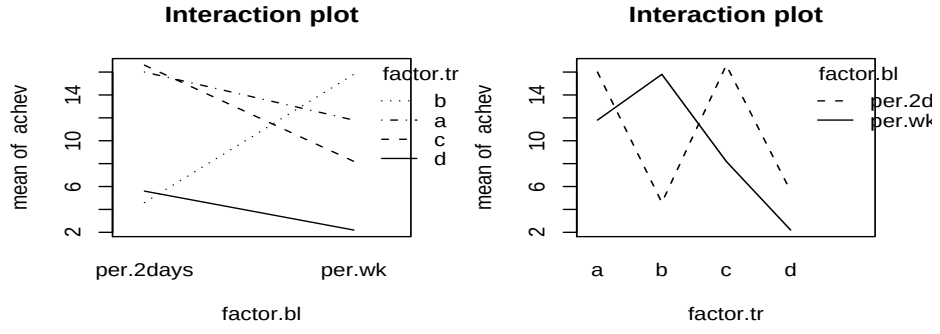
	Drug mixture				Bl. mean $\bar{y}_{i\cdot\cdot}$
Schedule	1	2	3	4	All mean $\bar{y}$
Per week	17	24	14	10	
	6	15	9	-1	
	10	10	12	0	
	12	16	0	3	
	14	14	6	-1	
Gp. mean $\bar{y}_{ij\cdot}$	11.8	15.8	8.2	2.2	9.5
Gp. var. s^2_{ij}	17.2	26.2	30.2	21.7	
Per two days	18	-2	20	-2	
	9	0	16	7	
	17	17	12	10	
	21	2	17	6	
	15	6	18	7	
Gp. mean $\bar{y}_{ij\cdot}$	16.0	4.6	16.6	5.6	10.7
Gp. var. s^2_{ij}	20.0	56.8	8.8	20.3	
Col. mean $\bar{y}_{\cdot j\cdot}$	13.9	10.2	12.4	3.9	10.1

$$\sum_{i=1}^{2}\sum_{j=1}^{4}\sum_{k=1}^{5} y^2_{ijk} = 6030$$

(a) Test for the effect of drug mixture on the improvement of headache index.

(b) Test for the effect of schedule on the improvement of headache index.

(c) Test for the interaction effect between drug mixture and schedule on the improvement of headache index. Comment the test result based on the following interaction plots.



(d) Complete the anova table.

Extra Practice Problems

1. A professor studied the effects of time ($i = 1, 2, 3$ for 9:00am, 1:00pm, 4:00pm) and seat configuration ($j = 1, 2$ for 'rows' or 'U-shape') on the student participation as measured by counting the number of times students asked or answered questions in 30 classes. Results were reported below:

	Rows ($j = 1$)			U-shape ($j = 2$)		
Replicate (k)	9:00am ($i = 1$)	1:00pm ($i = 2$)	4:00pm ($i = 3$)	9:00am ($i = 1$)	1:00pm ($i = 2$)	4:00pm ($i = 3$)
1	10	9	7	15	4	7
2	7	12	12	18	4	4
3	9	12	9	11	7	9
4	6	14	20	13	4	8
5	8	8	7	13	6	7
Group mean $\bar{y}_{ij\cdot}$	8	11	11	14	5	7
Group var. $s^2_{ij\cdot}$	2.5	6	29.5	7	2	3.5

   (a) Complete the ANOVA table.

   (b) Test if the number of times students asked or answered questions differs across seat configurations.

   (c) Test if the effects of time and seat configuration interact.