THE UNIVERSITY OF SYDNEY

STAT2012 Statistical Tests

Semester 2	**Solution to Tutorial Week 1**	2015

Tutorial questions

1. (a) From normal table,

$$\begin{aligned}
\Pr(-1.72 < Z < 0.52) &= \Pr(Z < 0.52) - \Pr(Z < -1.72) \\
&= 0.6985 - [1 - \Pr(Z < 1.72)] \\
&= 0.6985 - (1 - 0.9573) \\
&= 0.6985 - 0.0427 = 0.6558 \\
\Pr(|Z| > 1.96) &= 2\Pr(Z > 1.96) = 2[1 - \Pr(Z < 1.96)] \\
&= 2(1 - 0.975) = 0.05
\end{aligned}$$

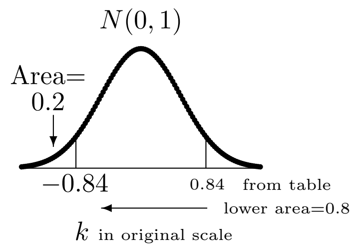
   (b) From normal table,

$$\begin{aligned}
\Pr(Z \le a) &= 0.7291 \Rightarrow a = 0.61 \\
\Pr(Z > b) &= 0.1 \Rightarrow \Pr(Z < b) = 0.9 \Rightarrow b = 1.28 \\
\Pr(|Z| < c) &= 0.90 \Rightarrow \Pr(Z < c) = 0.95 \Rightarrow c = 1.645
\end{aligned}$$

2. $X \sim \mathcal{N}(10, 16)$. From normal table,

$$\begin{aligned}
\Pr(X > 12) &= \Pr\left(Z > \frac{12-10}{4} = 0.5\right) = 1 - 0.6915 = 0.3085 \\
\Pr(X \le 7) &= \Pr\left(Z \le \frac{7-10}{4} = -0.75\right) = 1 - \Pr(Z < 0.75) = 1 - 0.7734 = 0.2266
\end{aligned}$$

$$\begin{aligned}
\Pr(X \le k) = &\quad 0.2 \\
\Rightarrow \quad \Pr(Z \le \tfrac{k-10}{4}) = &\quad 0.2 \\
\Rightarrow \quad \Pr(Z \le -\tfrac{k-10}{4}) = &\quad 0.8 \\
\Rightarrow \quad \tfrac{k-10}{4} = &\quad -0.84 \\
\Rightarrow \quad k = &\quad 10 + (-0.84) \times 4 = 6.64
\end{aligned}$$



3. We have $X_i \sim \mathcal{N}(20, 4^2)$, $i = 1, \ldots, 16$ (n is small).

   Then the sample mean being a linear function of normal r.v. is also normal, i.e. $\bar{X} \sim \mathcal{N}(20, 4^2/16)$, i.e. $\mathcal{N}(20, 1)$. Hence $\bar{X} - 20$ is standard normal $\bar{X} - 20 \sim \mathcal{N}(0, 1)$.

$$\Pr(\bar{X} \ge 22) = \Pr\left(Z \ge \frac{22-20}{1}\right) = \Pr(Z > 2) = 1 - 0.9772 = 0.0228$$

4. Let $X_1, X_2, ..., X_{10}$ be a random sample of size 10 from a $\mathcal{N}(15, 2.53^2)$ distribution.

$$\Pr\left(\sum_{i=1}^{10} X_i \le 158\right) = \Pr(\bar{X} \le 15.8) = \Pr\left(Z \le \frac{15.8 - 15}{2.53/\sqrt{10}}\right) = \Pr(Z \le 1) = 0.8413$$

5. (a) From t-table,

$$\begin{aligned} \Pr(t_3 < 2.353) &= 1 - 0.05 = 0.95 \\ \Pr(|t_{11}| > 2.718) &= 2(0.01) = 0.02 \end{aligned}$$

(b) From t-table,

$$\begin{aligned} \Pr(t_{20} \leq a) &= 0.1 \Rightarrow a = -1.325 \\ \Pr(|t_{16}| > b) &= 0.05 \Rightarrow \Pr(t_{16} > b) = 0.025 \Rightarrow b = 2.12 \\ \Pr(t_{10} > c) &= 0.90 \Rightarrow \Pr(t_{10} < -c) = 0.10 \Rightarrow c = -1.372 \end{aligned}$$

6. If $X_1, X_2, ..., X_n$ (n is small) be r.v. such that $X_i \sim \mathcal{N}(\mu, \sigma^2)$ where σ^2 is unknown. Then $T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$ where S is the sample standard deviation.

7. We have $X_i \sim \mathcal{N}(\mu, \sigma^2)$, $i = 1, \ldots, n$, $n = 25$, $\mu = 50$, $\bar{x} = 48$ and $s = 4$. Then the test statostic is

$$t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{48 - 50}{4/\sqrt{25}} = -2.5$$

where the d.f. is $n - 1 = 25 - 1 = 24$. Then

$$p - \text{value} = \Pr(t_\nu < t_0) = \Pr(t_{24} < -2.5) = 0.0098 < 0.01$$