THE UNIVERSITY OF SYDNEY
STAT2012 STATISTICAL TESTS

Semester 2	**Solution to Tutorial Week 2**	2015

Tutorial questions

1. (e) t_{15}

2. (c) 1.83, $\quad t_0 = \dfrac{\bar{y}-\mu_0}{S/\sqrt{n}} = \dfrac{63-60}{9/\sqrt{30}} = 1.8257$

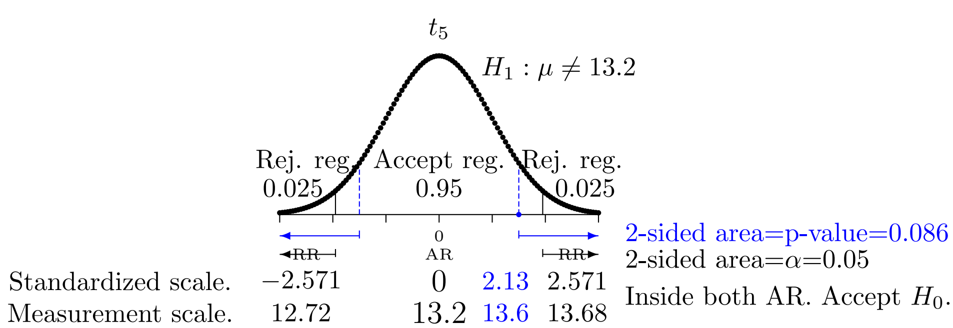
3. (d) one-sided t test and calculate the P-value using $P(t_{29} > 1.83)$

4. (b) 2.0, $\quad z_0 = \dfrac{7-6}{2/\sqrt{16}} = 2$

5. (c) the p-value is 0.0228 and we have strong evidence against H_0.

6. (a) We have $n = 6$, $\bar{y} = 13.6$, $s = 0.46$ and $\mu_0 = 13.2$. The t test for the mean amount of sales in a drug store last week is

1. **Hypotheses:**	H_0: $\mu = 13.2$ vs H_1: $\mu \neq 13.2$
2. **Test statistic:**	$t_0 = \dfrac{\bar{y}-\mu_0}{s/\sqrt{n}} = \dfrac{13.6-13.2}{0.46/\sqrt{6}} = 2.13$ or $\bar{y} = 13.6$.
3. **Assumption:**	$Y_i \sim \mathcal{N}(\mu, \sigma^2)$. Since σ^2 is unknown, $t_0 \sim t_{n-1}$.
4. **P-value:**	$0.05 < p\text{-value} = 2\Pr(t_5 > 2.13) < 0.10$ (0.0864)
4'. **Rejection region:**	$t_0 \leq -t_{5,0.025} = -2.571$ or $t_0 \geq t_{5,0.025} = 2.571$
4". **Rejection region:**	$\bar{y} \leq \mu_0 - t_{5,0.025}s/\sqrt{n}$ or $\bar{y} \geq \mu_0 + t_{5,0.025}s/\sqrt{n}$ i.e. $\bar{y} \leq 13.2 - 2.571 \cdot 0.46/\sqrt{6} = 12.7172$ or $\bar{y} \geq 13.2 + 2.571 \cdot 0.46/\sqrt{6} = 13.6828$
5. **Decision:**	Since p-value > 0.05, we do not reject H_0 or since $t_0 = 2.108 \in (-2.571,\ 2.571)$ in AR, we do not reject H_0 or since $\bar{y} = 13.6 \in (12.7172,\ 13.6828)$ in AR, we do not reject H_0 and conclude that the data are consistent with H_0 that the usual sales volume is \$13,200.



(b) The z test for the mean amount of sales in a drug store last week is

1. **Hypotheses:** H_0: $\mu = 13.2$ vs H_1: $\mu \neq 13.2$
2. **Test statistic:** $z_0 = \dfrac{\bar{y} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{13.6 - 13.2}{0.5/\sqrt{6}} = 1.9596$ or $\bar{y} = 13.6$.
3. **Assumption:** $Y_i \sim \mathcal{N}(\mu, \sigma^2)$. Since σ^2 is known, $z_0 \sim \mathcal{N}(0, 1)$.
4. **P-value:** p-value $= 2\Pr(Z > 1.96) = 0.05$

4'. **Rejection region:** $z_0 \leq -z_{0.025} = -1.96$ or $z_0 \geq z_{0.025} = 1.96$

4". **Rejection region:** $\bar{y} \leq \mu_0 - z_{0.975}\sigma/\sqrt{n}$ or $\bar{y} \geq \mu_0 + z_{0.975}\sigma/\sqrt{n}$
i.e. $\bar{y} \leq 13.2 - 1.96 \cdot 0.5/\sqrt{6} = 12.8$
or $\bar{y} \geq 13.2 + 1.96 \cdot 0.5/\sqrt{6} = 13.6$

5. **Decision:** Since p-value $\leq \alpha = 0.05$, we reject H_0 or since $z_0 = 1.96 \geq 1.96$ in RR, we reject H_0 or since $\bar{y} = 13.6 \geq 13.6$ in RR, we reject H_0. There is evidence in the data against H_0 that the usual sales volume is $13,200.

Note that RR is rejection region and AR is acceptance region.

Extra problems

1. Q7.4.21 (p405 of Larsen and Marx)
The t test for the mean pit depth with plastic coding is

   1. **Hypotheses:** H_0: $\mu = 0.0042$ vs H_1: $\mu < 0.0042$
   2. **Test statistic:** $t_0 = \dfrac{\bar{y} - \mu_0}{s/\sqrt{n}} = \dfrac{0.0039 - 0.0042}{0.00383/\sqrt{10}} = -0.247698$
   3. **Assumption:** $Y_i \sim \mathcal{N}(\mu, \sigma^2)$. Since σ^2 is unknown, $t_0 \sim t_{n-1}$.
   4. **P-value:** p-value $= \Pr(t_9 \leq -0.2477) > 0.05$ (0.4046899)
   5. **Decision:** Since p-value > 0.05, we accept H_0. The data is consistent with H_0 that the mean pit depth with plastic coding is 00042.

2. Q6.2.1 (p359 of Larsen and Marx)

   (a) Reject H_0 if $z_0 = \dfrac{\bar{y} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{114.2 - 120}{18/\sqrt{25}} = -1.61$. p-value=0.054$<$ 0.08, reject H_0.

   (b) Reject H_0 if $z_0 = \dfrac{\bar{y} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{45.1 - 42.9}{3.2/\sqrt{16}} = 2.75$. p-value=0.00596$<$ 0.01, reject H_0.

   (c) Reject H_0 if $z_0 = \dfrac{\bar{y} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{15.8 - 14.2}{4.1/\sqrt{9}} = 1.17$. p-value=0.121$<$ 0.13, reject H_0.

3. Q6.2.7 (p360 of Larsen and Marx)
The z test for the mean is

1. **Hypotheses:** H_o: $\mu = 12.6$ vs H_1: $\mu \neq 12.6$
2. **Test statistic:** $z_0 = \dfrac{\overline{y} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{12.76 - 12.6}{0.4/\sqrt{30}} = 2.19$
3. **Assumption:** $Y_i \sim \mathcal{N}(\mu, \sigma^2)$. Since σ^2 is known, $z_0 \sim \mathcal{N}(0, 1)$.
4. **P-value:** p-value $= 2\,\Pr(Z > 2.19) = 0.0285$
5. **Decision:** Since p-value < 0.05, we reject H_0. There is evidence against H_0 that the mean is 12.6.

4. The t test for the mean is

1. **Hypotheses:** H_0: $\mu = 50$ vs H_1: $\mu \neq 50$
2. **Test statistic:** $t_0 = \dfrac{\overline{y} - \mu_0}{s/\sqrt{n}} = \dfrac{48 - 50}{4/\sqrt{25}} = -2.5$
3. **Assumption:** $Y_i \sim \mathcal{N}(\mu, \sigma^2)$. Since σ^2 is unknown, $t_0 \sim t_{n-1}$.
4. **P-value:** $0.01 < p\text{-value} = 2\,\Pr(t_{24} < -2.5) < 0.02$ (0.019654)
5. **Decision:** Since p-value < 0.05, we reject H_0. There is evidence against H_0 that the mean is 50.