THE UNIVERSITY OF SYDNEY

STAT2012 STATISTICAL TESTS

Semester 2	**Solution to Tutorial Week 9**	2015

Tutorial questions

1. The two-way ANOVA tests for treatment and/or block and/or interaction effects are:

   (a) **Hypotheses:**

$$
\begin{array}{llll}
H_0: & \beta_1 = \beta_2 = ... = \beta_4 = 0 & \text{vs} & H_1: \text{ Not all } \beta_j \text{ are same;} \\
H_0: & \alpha_1 = \alpha_2 = 0 & \text{vs} & H_1: \text{ Not all } \alpha_i \text{ are same.} \\
H_0: & \delta_{ij} = 0, \; i = 1,2; \; j = 1,...,4 & \text{vs} & H_1: \text{ Not all } \delta_{ij} \text{ are same.}
\end{array}
$$

   (b) **Test statistic:** For treatment, block and interaction effects respectively:

$$
\begin{aligned}
f_{t0} &= \frac{SST/(c-1)}{SSR/rc(m-1)} = \frac{581.8/3}{804.8/(2(4)4)} = 7.7111 \\
f_{b0} &= \frac{SSB/(r-1)}{SSR/rc(m-1)} = \frac{14.4/1}{804.8/(2(4)4)} = 0.5726 \\
f_{i0} &= \frac{SSI/(r-1)(c-1)}{SSR/rc(m-1)} = \frac{548.6/3}{804.8/(2(4)4)} = 7.2710
\end{aligned}
$$

   where

$$
\begin{aligned}
CM &= rcm\,\bar{y}^2 = 2(4)5(10.1^2) = 4080.4 \\
SST_o &= \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{m} {y_{ijk}}^2 - rcm\,\bar{y}^2 \\
&= 6030 - 4080.4 = 1949.6 \\
SST &= rm\sum_{j=1}^{c} \bar{y}_{.j.}^2 - rcm\,\bar{y}^2 \\
&= 2(5)(13.9^2 + \cdots + 3.9^2) - 4080.4 = 581.8 \\
SSB &= cm\sum_{i=1}^{r} \bar{y}_{i..}^2 - rcm\,\bar{y}^2 \\
&= 4(5)(9.5^2 + 10.7^2) - 4080.4 = 14.4 \\
SSR &= (m-1)\sum_{i=1}^{r}\sum_{j=1}^{c} s_{ij.}^2 \\
&= 4(17.2 + \cdots + 20.3) = 804.8 \\
SSI &= SST_o - SSB - SST - SSR \\
&= 1949.6 - 581.8 - 14.4 - 804.8 = 548.6
\end{aligned}
$$

(c) **Assumption:** $Y_{ijk} \sim \mathcal{N}(\mu_{ij}, \sigma^2)$, $\mu_{ij} = \mu + \alpha_i + \beta_j + \delta_{ij}$ and Y_{ijk} are independent.

(d) **P-value:** For treatment, block and interaction effects respectively:

p-value $= \Pr(F_{3,32} \geq 7.7111) < 0.001$ ($F_{3,30,0.999} = 7.05$; 0.0091 from R)

p-value $= \Pr(F_{1,32} \geq 0.5726) > 0.1$ ($F_{1,30,0.900} = 2.88$; 0.6372 from R)

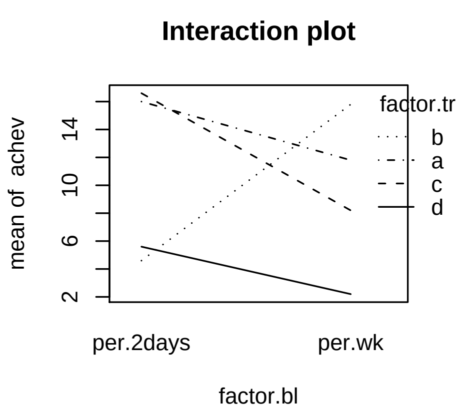
p-value $= \Pr(F_{3,32} \geq 7.2710) < 0.001$ ($F_{3,30,0.999} = 7.05$; 0.0007 from R)

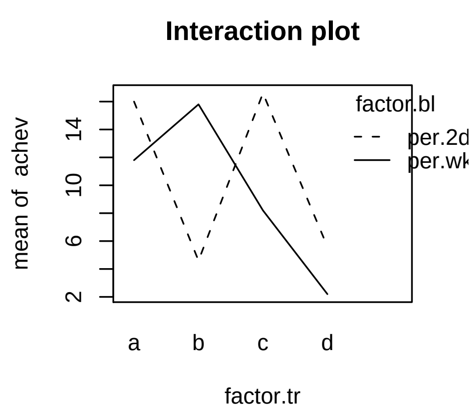
(e) **Decision:** we reject H_0 of no treatment and interaction effects. There are strong evidence in the data that improvement in headache index differ across drug mixture. There are also strong evidence in the data that interaction effect exists between drug mixture and schedule. However the data is consistent with the null hypothesis that improvement in headache index is the same for the two schedules.

Two way ANOVA table for factorial design

Source	**df**	**SS**	**MS**	**F**
Treatments	3	581.8	$\frac{581.8}{3} = 193.933$	$\frac{193.933}{25.15} = 7.7111$
Blocks	1	14.4	$\frac{14.1}{1} = 14.1$	$\frac{14.1}{25.15} = 0.5726$
Interactions	3	548.6	$\frac{548.6}{3} = 182.8667$	$\frac{182.8667}{25.15} = 7.2710$
Residuals	32	804.8	$\frac{804.8}{32} = 25.15$	
Total	39			

The interaction effect can be easily seen from the following interaction plots as there are many crossovers which indicate inconsistency of the effects from one factor across the levels of the other factor.





Extra problems

1. Two-way factorial data:

   (a) We have $n = 30$, $r = 3$, $c = 2$, $m = 5$, $j = 1, 2$ to indicate seat configuration and $i = 1, 2, 3$ to indicate times.

$$\overline{y}_{.1.} = 10,\ \overline{y}_{.2.} = 8.\dot{6},\ \sum_j \overline{y}_{.j.}^2 = 175.\dot{1},\ \sum_{i,j} \overline{y}_{ij.}^2 = 576,\ \sum_{i,j} s_{ij}^2 = 50.5$$

$$\overline{y}_{1..} = 11,\ \overline{y}_{2..} = 8,\ \overline{y}_{3..} = 9,\ \sum_i \overline{y}_{i..}^2 = 266,\ \sum_{i,j,k} y_{ijk}^2 = 3082,\ \overline{y}_{...} = 9.\dot{3}$$

$$\begin{aligned}
SST_o &= \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{m} y_{ijk}^2 - n\overline{y}^2 = 3082 - 30(9.\dot{3})^2 = 468.\dot{6} \\
SSB &= cm\sum_{i=1}^{r} \overline{y}_{i..}^2 - n\overline{y}^2 = 2(5)(266) - 30(9.\dot{3})^2 = 46.\dot{6} \\
SST &= rm\sum_{j=1}^{c} \overline{y}_{.j.}^2 - n\overline{y}^2 = 3(5)(175.\dot{1}) - 30(9.\dot{3})^2 = 13.\dot{3} \\
SSR &= (m-1)\sum_{i=1}^{r}\sum_{j=1}^{c} s_{ij}^2 = (4)(50.5) = 202 \\
SSI &= SST_o - SST - SSB - SSR = 468.\dot{6} - 13.\dot{3} - 46.\dot{6} - 202 = 206.\dot{6}
\end{aligned}$$

ANOVA table	df	SS	MS	F	p-value	critical value
Seat conf.	1	$13.\dot{3}$	$13.\dot{3}$	1.584158	0.220269	4.259675
Time	2	$46.\dot{6}$	$23.\dot{3}$	2.772277	0.082567	3.402832
Interaction	2	$206.\dot{6}$	$103.\dot{3}$	12.27723	0.000213	3.402832
Residual	24	202	$8.41\dot{6}$			
Total	29	$468.\dot{6}$				

   (b) The F-test is

   1. **Hypothesis:** H_0: $\beta_1 = \beta_2 = 0$ vs H_1: They are not the same.
   2. **Test statistic:** $f_0 = \dfrac{MST}{MSR} = 1.5842$
   3. **Assumption:** $Y_{ijk} \sim \mathcal{N}(\mu_{ij}, \sigma^2)$, $\mu_{ij} = \mu + \alpha_i + \beta_i + \delta_{ij}$, Y_{ijk} are independent.
   4. **P-value:** $\Pr(F_{1,24} \geq 1.5842) > 0.1 \quad (F_{1,24,0.9} = 2.93)$
   5. **Conclusion:** Do not reject H_0. The data is consistent with the null hypothesis that the seat configuration effects are the same.

   (c) The F test is

1. **Hypothesis:** H_0: the 2 factors not interact vs H_1: the 2 factors interact
2. **Test statistic:** $f_0 = \dfrac{SSI}{SSR} = 12.2772$
3. **Assumption:** $Y_{ijk} \sim \mathcal{N}(\mu_{ij}, \sigma^2)$, $\mu_{ij} = \mu + \alpha_i + \beta_i + \delta_{ij}$, Y_{ijk} are independent.
4. **P-value:** $\Pr(F_{2,24} \geq 12.2772) < 0.001 \quad (F_{2,24,0.999} = 9.34)$
5. **Conclusion:** Reject H_0. There is strong evidence in the data against the null hypothesis.